1. Find the derivatives of each of the following functions! Do not simplify!
   (a) \( f(x) = (\cos x)(\sin 2x) \)
   \[ f'(x) = -\sin x \sin 2x + 2 \cos x \cos 2x. \]
   (b) \( g(\theta) = \sqrt{2\theta^3 - \theta - 1} + 3.5(18\theta^4)^{-4.14} - \theta \)
   \[ g'(\theta) = \frac{6\theta^2 - 1}{2\sqrt{2\theta^3 - \theta - 1}} - 4.14 \cdot 3.5(18\theta^4)^{-5.14}2\theta^3 - 1 \]
   (c) \( y = \frac{x^{3/7}}{x^2 - 3x - 1} \)
   \[ y' = \frac{\frac{3}{7}x^{-4/7}(x^2 - 3x - 1) - x^{3/7}(2x - 3)}{(x^2 - 3x - 1)^2} \]
   (d) \( y = \sqrt{3 + \tan x} \)
   \[ y' = \frac{\sec^2 x}{2\sqrt{3 + \tan x}} \]
   (e) \( y = \sqrt{\sin(3\pi x)} \)
   (f) \( y' = \frac{3\pi \cos(3\pi x)}{2\sqrt{\sin(3\pi x)}} \)
   (g) \( f(x) = x\sqrt{2 + x^2} \)
   \[ f'(x) = \sqrt{2 + x^2} + \frac{x^2}{\sqrt{2 + x^2}} = \frac{2(1 + x^2)}{\sqrt{2 + x^2}} \]

2. Differentiate \( f(x) = \frac{1}{\pi x^2} \) using first principles. No work, no credit.
   \[ f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a} \frac{\frac{1}{\pi x^2} - \frac{1}{\pi a^2}}{x - a} = \lim_{x \to a} \frac{\frac{a^2 - x^2}{\pi x^2 a^2(x - a)}}{x - a} = \lim_{x \to a} \frac{-(a + x)}{\pi x^2 a^2} = -\frac{2}{\pi a^3} \]

3. Find all asymptotes of the function \( f(x) = \frac{x^3 + 8}{x^2 - 1} \) and sketch the graph of \( f(x) \). Rewrite the \( f(x) \) as
   \[ f(x) = \frac{(x + 2)(x^2 - 2x + 4)}{(x - 1)(x + 1)} = x + \frac{x + 8}{x^2 - 1}. \]
   There will be vertical asymptotes at \( x = 1 \) and \( x = -1 \), and there is a slant asymptote \( l(x) = x \). There is an \( x \)-intercept at \( x = -2 \) and a \( y \)-intercept at \( -8 \). This should help you to sketch a graph.
4. (15) Compute each of the following limits or show that they do not exist. **Show your work!**

(a) \( \lim_{x \to -3} \frac{x + 3}{x^2 + 7x + 12} = \lim_{x \to -3} \frac{x + 3}{(x + 3)(x + 4)} = 1 \)

(b) \( \lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} = \lim_{x \to 9} \frac{\sqrt{x} - 3}{(\sqrt{x} - 3)(\sqrt{x} + 3)} = \frac{1}{6} \)

(c) \( \lim_{x \to 0} \frac{x}{\sin 3x} = \frac{1}{3} \lim_{x \to 0} \frac{3x}{\sin 3x} = \frac{1}{3} \)

5. (15) Find the equation of the tangent line to the curve \( x^3 + y^3 = 9xy \) at the point \((4, 2)\). **Show your work!**

We differentiate the equation for the curve with respect to the variable \( x \), and then we substitute the values \( x = 4 \) and \( y = 2 \):

\[
3x^2 + 3y^2 \frac{dy}{dx} = 9y + 9x \frac{dy}{dx} \quad \text{and} \quad \frac{dy}{dx} = \frac{5}{4}.
\]

The equation for the tangent line, in point–slope form, will be

\[
t(x) = \frac{5}{4} (x - 4) + 2.
\]

As approximate value for \( y \) when \( x = 4.1 \) we find

\[
y(4.1) \sim t(4.1) = \frac{5}{4} \cdot \frac{1}{10} + 2 = 2.125
\]

6. (15) 100m\(^3\) of oil is spilled when a tanker collides with a tuna boat. The resulting oil slick forms a right circular cylinder on the surface of the water. If the thickness (\( h \)) of the slick is decreasing at a rate of 0.001 m/sec, how fast is the radius (\( r \)) increasing when the slick is 0.01 m thick? Note: \( V = \pi r^2 h \).

We differentiate the volume formula with respect to time. Here we observe that \( h \) and \( r \) both depend on time, and that \( V \) does not. Hence

\[
\frac{dV}{dt} = 0 = \pi \left( 2r \frac{dr}{dt} h + r^2 \frac{dh}{dt} \right) \quad \text{which simplifies to} \quad \frac{dr}{dt} = -\frac{r}{2h} \frac{dh}{dt}.
\]

We calculate \( r \) at the moment when \( h = .01 \) and find \( r = 100/\sqrt{\pi} \). After substituting this value for \( r \) as well as \( h = .01 \) and \( dh/dt = -0.001 \) we find \( dr/dt = 5/\sqrt{\pi} \sim 2.8 \). This means that the radius of the oil slick increases at a rate of about 2.8 m/sec.