

## Calculus I (Math 241) – Test 2

(No Work – No Credit)

**Problem 1.** [15 Points] You have 300 meters of fencing material, and you like to fence off a rectangular corral. On one side of the corral is a cliff, and no fence is needed. How long should the sides of the corral be so that its area is maximal? Give an argument why your solution is in fact optimal.

**Problem 2.** [15 Points] Consider a function  $y = y(t)$  that satisfies the differential equation

$$y' = y(100 - y).$$

1. Apparently,  $y'$  depends on  $y$  and not on  $t$ . In which range for  $y$  does  $y$  increase, and in which ones does it decrease.
2. Use implicit differentiation and work out an expression for  $y''$  in terms of  $y$ . In which intervals for  $y$  is  $y$  concave up, respectively concave down?

**Problem 3.** [5 Points] State Cauchy's Mean Value Theorem. You need to specify the assumptions and state the assertion.

**Problem 4.** [10 Points] Suppose that the function  $f(x)$  is continuous on the interval  $[a, b]$ , and that it is differentiable on  $(a, b)$ . Suppose that  $f(a) = f(b) = 0$ . Show that  $f(x)$  has a critical point in  $(a, b)$ .

**Problem 5.** [20 Points] Construct an open box (it has a bottom but no lid), whose base is a rectangle that is twice as long as wide. What should its dimensions be (length, width, and height) if you are using 600 square centimeter of material and its volume is maximal?

**Problem 6.** [35 Points] Discuss the function

$$f(x) = (x^2 - 4)(x + 1).$$

More specifically:

1. Find the intercepts of  $f$  with the coordinate axes and the intervals on which the function  $f$  is positive, respectively negative.
2. Find the critical points of the function and the intervals on which  $f$  is increasing, respectively decreasing.

3. Use the values of  $f$  and  $f'$  at  $x = 1$  to get an approximate value for  $f(1.2)$ .
4. Find the inflection points and the intervals on which  $f$  is concave up, respectively concave down.
5. Find where the local minima and maxima of the function  $f$  occur.
6. Sketch the graph of  $f$  based on the information that you obtained.