INSTRUCTIONS: Write legibly. Indicate your answer clearly. Show all work; explain your answers. Answers with work not shown might be worth zero points. No calculators, cell phones, or cheating.

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(6) Extra Credit: Show that for all $x$ and $y$, $|\sin(x) - \sin(y)| \leq |x - y|$. 
(20) 1. Find the derivative; do not simplify your answer.

(a) \( f(x) = \frac{\sqrt{x}}{x^3 + 1} \)

\[ f'(x) = \]

(b) \( g(x) = \sec \left( 2x + \frac{\pi}{4} \right) \)

\[ g'(x) = \]

(c) \( h(x) = \int_{\sin x}^{x} \tan^2(t + 1) \, dt \)

\[ h'(x) = \]

(d) \( k(x) = \tan(2x)\sqrt{1 + \cos(3x)} \)

\[ k'(x) = \]
2. Differentiate $f(x) = x^3 - 3x$ at $x = 2$ using first principles (find the limit of the appropriate difference quotient). No credit for the use of the rules of differentiation!

3. Find the equation of the tangent line to the graph of the function $f(x) = \tan x$ at $x = \pi/3$. 

(8) 2. Differentiate $f(x) = x^3 - 3x$ at $x = 2$ using first principles (find the limit of the appropriate difference quotient). No credit for the use of the rules of differentiation!

(6) 3. Find the equation of the tangent line to the graph of the function $f(x) = \tan x$ at $x = \pi/3$. 

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4. Consider the curve $C$ given by the equation $\sin y = \cos \sqrt{x}$.

(a) What is the slope of the curve $C$ at the point $P = (\pi^2/4, 0)$?

(b) Let $Q$ be another point on the curve whose $x$–coordinate is $\pi^2/4 + .1$. Find an approximate value for the $y$–coordinate of $Q$.

(c) Thinking of $y$ as an implicitly defined function of $x$, find $y''$ at the point $P$.

5. Evaluate the limit, or explain why it does not exist:

(a) $\lim_{x \to 1} \frac{x - 1}{x^4 - 1}$

(b) $\lim_{x \to \infty} x \left( x - \sqrt{x^2 + 1} \right)$
6. Give a well supported argument that every polynomial of degree three has a real root.

7. Consider a point \( P = (x, y) \) that moves along the graph of the function \( y = \frac{8}{x} \) with a horizontal velocity of 3 units per second. At which rate does the distance between \( P \) and the origin \((0,0)\) of the coordinate system change as the point passes through \((4,2)\)?
8. Compute the integrals:

(a) \[ \int x(x + 2)^{15} \, dx = \]

(b) \[ \int \tan^2(2x) \, dx = \]

(c) \[ \int |x| \, dx = \]

9. Sketch and find the area of the region that lies above the \( x \)-axis and below both parabolas, \( p = 4 - (x - 2)^2 \) and \( q = 4 - (x - 4)^2 \).
Let \( f(x) = x(x - 3)^2 = x^3 - 6x^2 + 9x \).

(a) Find the zeros of \( f(x) \) and provide the intervals on which \( f(x) \) is positive, respectively negative.

(b) \( f'(x) = \) \hspace{2cm} \( f''(x) = \)

(c) Find the critical points and intervals on which \( f \) is increasing/decreasing.

(d) Find the local extrema.

(e) Find the absolute maxima and minima of \( f \) on \([-1, 3.9]\)
(f) Find the inflection points and the intervals on which \( f(x) \) is concave, respectively down.

(g) Sketch the graph of \( f(x) \).
11. Find the radius $r$ and the height $h$ of a right circular cylinder if its volume is maximal and it fits inside a hemisphere of radius 15. What is the ratio $h/r$?

12. We like to compute the Riemann sum for the function $f(x) = x^2$ on the interval $I = [1, 2]$. Specifically, partition $I$ into four intervals of equal lengths, and use their midpoints as distinguished points in the computation.
(12) 13. Let $\Omega$ be the region bounded by the $x$-axis $y = 0$ and parabola $p(x) = 1 - (x - 2)^2$. Set up definite integrals representing each of the following quantities (DO NOT EVALUATE YOUR INTEGRALS!):

(a) the volume of the solid obtained by revolving $\Omega$ around the $x$-axis.

(b) the volume of the solid obtained by revolving $\Omega$ around the $y$-axis. Use the shell method.

(c) the volume of the solid obtained by revolving $\Omega$ around the $y$-axis. Use the washer method.