Problem 1. [12 Points] Calculate the derivatives (do not simplify):
\[
\frac{d}{dx} (x^3 \cos^2(3x + 1)) \quad \text{and} \quad \frac{d}{dx} \left[ \frac{x^2 + 1}{(x^3 + x)^4} \right] \quad \text{and} \quad \frac{d}{dx} \arctan(x^2 - \sin x)
\]

Problem 2. [12 Points] Calculate the integrals:
\[
\int_0^{\pi/2} \sin(3x) \, dx \quad \text{and} \quad \int t(t + 1)^9 \, dt \quad \text{and} \quad \int \frac{x}{(x^2 + 4)^2} \, dx.
\]

Problem 3. [6 Points] Define the concept of a limit. In other words, make precise the statement that
\[
L = \lim_{{x \to a}} f(x)
\]
for a given function \(f(x)\) and an interior point \(a\) of its domain.

Problem 4. [12 Points] Find the equation of the tangent line to the function \(f(x) = x^4 - 7x^2\) at \(x = 1\), and use approximation by differentials to find an approximate value of \(f\) at \(x = 1.05\).

Problem 5. [24 Points] Consider a round box with top and bottom. Its interior is divided into quarters. It is made from \(S\) square centimeters of material. Find its height \((h)\), radius \((r)\), and the ratio \(h/r\) if its volume is maximal.

Problem 6. [12 Points] The third root of 9 is a zero of the function \(f(x) = x^3 - 9\), and a fairly good first guess for it is \(x_0 = 2\). Use Newton’s method once to improve this guess.

Problem 7. [12 Points] Consider a curve \(y(x)\) that satisfies
\[
\frac{dy}{dx} = \frac{y}{x^2} \quad \text{with} \quad y(1) = 3.
\]
Decide whether \(y\) is concave up or down at \((x, y) = (1, 3)\).

Problem 8. [6 Points] State the Fundamental Theorem of Calculus. This includes the assumptions and conclusion.
Problem 9. [30 Points] Discuss the function

\[ f(x) = x(x - 3)^2 = x^3 - 6x^2 + 9x. \]

Specifically, answer the following questions:

1. On which intervals is the function positive, resp., negative.
2. On which intervals is the function increasing, resp., decreasing.
3. On which intervals is the function concave up, resp., concave down.
4. Find the critical points of the function and decide whether they are local maxima or minima.
5. Find the inflections points of the function.
6. Find the absolute extrema of the function on the interval \([-1, 2.5]\).
7. Sketch the graph in accordance with the information obtained above.

Problem 10. [12 Points] In this problem you are expected to calculate a Riemann sum. The function is \( f(x) = x^2 \), and the interval is \([0, 3/2]\). Partition the interval into three subintervals of equal length, and choose the midpoint in each of the subintervals as distinguished point.

Problem 11. [12 Points] Sketch the region between the curves \( y = 1 \) and \( y = 1 + \sin x \) for \( x \in [0, \pi] \). Compute the volume of the solid of revolution if the region is revolved around the \( x \)-axis.

Problem 12. [3 Points Extra Credit for each part]

1. Repeat the previous problem, but use the interval \([0, 3\pi/2]\).
2. Find the volume of the solid if the region in the previous problem is revolved around the \( y \)-axis. Express your answer as an integral, without evaluating it.