Calculus I (Math 241) – Final

Problem 1. [16 Points] Calculate the derivatives of the following functions:

$$f(x) = \tan^2 x, \quad g(x) = \sqrt{1+\cos(2x)}, \quad h(x) = 5x\sec(x^2), \quad t(x) = \frac{1}{1+x^2}.$$

Problem 2. [16 Points] Calculate the integrals:

$$\int_0^{\frac{\pi}{3}} \cos(2x)\ dx, \quad \int \frac{x^2}{\sqrt{x+2}}\ dx, \quad \int \frac{dt}{\sqrt{4-t^2}}, \quad \int_0^{\frac{\pi}{3}} \cos^2 x\ dx.$$

Problem 3. [25 Points] Discuss the function

$$f(x) = (x-2)(x+1)^2 = x^3 - 3x - 2.$$

Specifically, address the following:

1. Find the intercepts and the intervals on which the function is positive, resp., negative.
2. Find the critical points and the intervals on which the function is increasing, resp., decreasing.
3. Find the inflection points and the intervals on which the function is concave up, resp., concave down.
4. Find the local maxima and minima.
5. Find the absolute extrema of the function on the interval $[-1.3, 3]$.
6. Sketch the graph in accordance with the information obtained above.

Problem 4. [15 Points] Use first principles to find $f'(a)$ if $f(x) = 1/\sqrt{x}$.

Problem 5. [12 Points] Consider a curve that is implicitly defined by the equation

$$2\sin\left(\frac{\pi}{12}(x^2+y)\right) = x.$$

Find an implicit equation for y', and find the equation for the tangent line to the curve at the point $(x, y) = (1, 9)$.

Problem 6. [20 Points] Consider an open box. Its base is a square, it has a bottom and no top, and it has a diagonal divider. It is made from S square centimeters of material. Find the height (h), side length (a), and the ratio h/a for its volume to be maximal.

Problem 7. [10 Points] State the Intermediate Value Theorem, and use it to show that $y = x^3 - 4x^2 + x + 1$ has a root between $x = 0$ and $x = 1$. Make sure that you state why the assumptions of the theorem hold, and how its conclusion implies the desired assertion about y.

Problem 8. [6 Points] State Cauchy's Mean Value Theorem. This includes the assumptions and conclusion.

Problem 9. [12 Points] In this problem you are expected to calculate a Riemann sum. The function is $f(x) = 1/x$, and the interval is $[1, 5/2]$. Partition the interval into three subintervals of equal length, and choose the midpoint in each of the subintervals as distinguished point.

Problem 10. [18 Points] Sketch the region Ω in the 1st quadrant between the curves $y = (3x)/(5\pi)$ and $y = \sin x$.

1. Find the area of the region Ω.
2. Find the volume of the solid of revolution if Ω is revolved around the x-axis.
3. Set up an integral that computes the volume of the solid of revolution if Ω is revolved around the y-axis.