Calculus II (Math 242) – Test 1
(No Work – No Credit)

Problem 1. [4 Points] Define the natural logarithm function $\ln x$.

Problem 2. [15 Points] Calculate the following limits:

$$(1)\ \lim_{x\to\infty} \frac{\ln x}{\sqrt{x}} \quad (2)\ \lim_{x\to 0} \frac{\sin^2 x}{\tan(x^2)} \quad \text{and} \quad (3)\ \lim_{x\to 0^-} (1+\sin(4x))^{\cot x}.$$

Problem 3. [6 Points] Graph and differentiate the function $\arccos(x)$. Important: Show your work!

Problem 4. [10 Points] Find the partial sum decomposition of

$$r(x) = \frac{x^2+x+1}{(x^2+1)(x+1)^2}.$$

Problem 5. [56 Points] Work out the following integrals:

$$(1)\ \int_0^1 \arctan x\ dx \quad (2)\ \int x\sin x\ dx \quad (3)\ \int \sec^4 x\ dx \quad (4)\ \int \cos^3 x\ dx$$

$$(5)\ \int \frac{\sqrt{x^2-9}}{x^4}\ dx \quad (6)\ \int_0^4 \frac{dx}{1+\sqrt{x}} \quad (7)\ \int \frac{2x}{x^2+2x+5}\ dx.$$

Problem 6. [7 Points] A population $P(t)$ grows exponentially, $P(2) = 200$, and $P(5) = 700$. Find the relative growth rate and the doubling time.

Problem 7. [10 Points] 500 grams of salt are dissolved in a container with 200 liters of water. The brine is diluted by adding less salty water, containing 1 gram per liter, at a rate of 10 liters per minute. The salt is evenly distributed in the water at any time, and water is drained from the container at the same rate as water is added. Set up an initial value problem whose solution is the concentration of salt $S(t)$ at any time t, and write down the solution of the problem.