HW 1, assigned 8/24

Based on geometric reasoning we asserted that

$$t(x) = -\frac{x_0}{\sqrt{r^2 - x_0^2}}(x - x_0) + \sqrt{r^2 - x_0^2}$$

is the tangent line to the circle $x^2 + y^2 = r^2$ through the point $(x_0, y_0) = (x_0, \sqrt{r^2 - x_0^2})$. Show

(1) The only intersection point of the line and circle is $(x_0, y_0) = (x_0, \sqrt{r^2 - x_0^2})$.

(2) Any line with a different slope than $t(x)$, through the point (x_0, y_0), will intersect the circle in two points: Say

$$\ell(x) = \left[-\frac{x}{\sqrt{r^2 - x_0^2}} + a\right](x - x_0) + \sqrt{r^2 - x_0^2}$$

and $a \neq 0$, then $\ell(x)$ and the circle have two intersection points.

P.S. In the above $y_0 > 0$ and $x_0 \neq \pm r$. Otherwise we need to modify the assertion.