Calculus I (Math 251A) – Test 1
(No Work – No Credit)

Problem 1. [10 Points] Find the equation for the tangent line to the graph of the function $f(x) = x^2 \cos x$ at the point $x = \pi/3$.

Problem 2. [10 Points] Differentiate $f(x) = \sqrt{x}$ using **first principles.**

Problem 3. [10 Points] Find the following limits:

$$\lim_{t\to 1} \frac{t^2+t-2}{t^2-1} \quad \text{and} \quad \lim_{x\to 4} \frac{4-x}{5-\sqrt{x^2+9}}.$$

Problem 4. [10 Points] Show that $f(x) = x^3 - \pi x + \sqrt{2}$ has a root between $x = 1$ and $x = 2$. **Provide a careful, complete argument.**

Problem 5. [10 Points] Use approximation by differentials to find an approximate value for $\sec 46°$.

Problem 6. [10 Points] A particle moves along the parabola $y = x^2$ in the first quadrant in such a way that the x–coordinate (measured in meters) increases at a steady 10 meters per second. How fast is the angle of inclination θ of the line joining the particle to the origin changing when $x = 3$ meters?

Problem 7. [10 Points] Find the slope of the curve $x^3 + y^3 - 9xy = 0$ (the folium of Descartes) at the point $(4, 2)$.

1.) $f'(x) = 2x\cos x - x^2 \sin x$

$f(\pi/3) = \frac{\pi^2}{9} \cdot \frac{1}{2}$ $\quad f'(\pi/3) = 2\frac{\pi}{3} \cdot \frac{1}{2} - \frac{\pi^2}{9}\frac{\sqrt{3}}{2}$

$t(x) = \frac{\pi}{3}\left(1 - \frac{\pi}{3}\frac{\sqrt{3}}{2}\right)\left(x - \frac{\pi}{3}\right) + \frac{\pi^2}{18} = \frac{6\pi - \pi^2\sqrt{3}}{18}\left(x - \frac{\pi}{3}\right) + \frac{\pi^2}{18}$

2.) $f'(a) = \lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x - a} = \lim_{x \to a} \frac{\cancel{\sqrt{x} - \sqrt{a}}}{\cancel{(\sqrt{x} - \sqrt{a})}(\sqrt{x} + \sqrt{a})} = \frac{1}{2\sqrt{a}}$

3.) $\lim_{t \to 1} \frac{t^2 + t - 2}{t^2 - 1} = \lim_{t \to 1} \frac{\cancel{(t-1)}(t+2)}{\cancel{(t-1)}(t+1)} = \frac{3}{2}$

$\lim_{x \to 4} \frac{4 - x}{5 - \sqrt{x^2+9}} = \lim_{x \to 4} \frac{(4-x)(5 + \sqrt{x^2+9})}{25 - x^2 - 9} = \lim_{x \to 4} \frac{(4-x)(5 + \sqrt{x^2+9})}{16 - x^2}$

$= \lim_{x \to 4} \frac{\cancel{(4-x)}(5 + \sqrt{x^2+9})}{\cancel{(4-x)}(4+x)} = \frac{5 + \sqrt{25}}{8} = \frac{5}{4}$

4.) a.) $f(1) = 1 - \pi + \sqrt{2} < 0$

b.) $f(2) = 8 - 2\pi + \sqrt{2} > 0$

c.) $f(x)$ is a polynomial, hence continuous.

d.) Zero is between $f(1)$ and $f(2)$. So zero is a value for some x between 1 and 2.

5.) $\sec'(x) = \sec x \tan x$

$\sec 45^\circ = \sec\frac{\pi}{4} = \sqrt{2}$

$\sec' 45^\circ = \sec'\frac{\pi}{4} = \sqrt{2}$

$\sec 46^\circ \sim \sec'\left(\frac{\pi}{4}\right)\left(\frac{\pi}{180}\right) + \sec\left(\frac{\pi}{4}\right) = \sqrt{2} \cdot \frac{\pi}{180} + \sqrt{2}$

$= \sqrt{2}\left(1 + \frac{\pi}{180}\right)$

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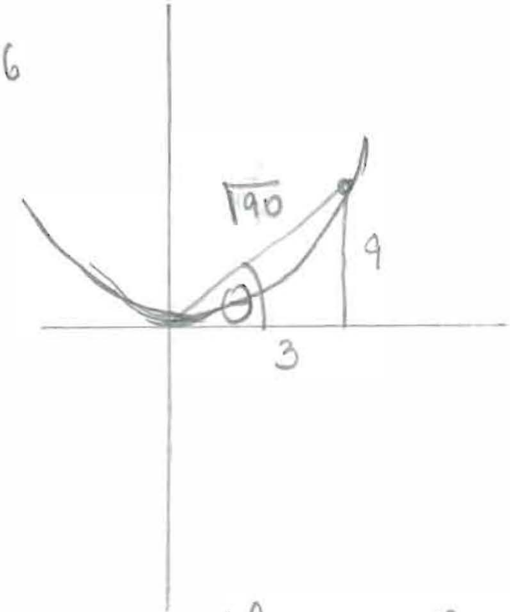

$$\tan\theta = \frac{y}{x} = x$$

x depends on t, and $\frac{dx}{dt} = 10$

θ depends on t

$$\frac{d}{dt}\tan\theta = \sec^2\theta \cdot \frac{d\theta}{dt} = 10$$

$$\frac{d\theta}{dt} = 10\cos^2\theta.$$

When $x=3$, then $\cos\theta = \frac{3}{\sqrt{90}}$. At the given moment:

$$\frac{d\theta}{dt} = \frac{90}{90} = 1$$

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$$3x^2 + 3y^2\, dy/dx - 9y - 9x\,y' = 0$$

substitute $x=4, y=2$:

$$48 + 12y' - 18 - 36y' = 0$$

$$30 = 24y' \quad \text{or} \quad y' = \frac{5}{4}$$