1. (20) Provide short answers:
   
   (a) Let $G$ be a group. What does it mean that $H$ is a subgroup of $G$?
   A nonempty subset $H$ of a group $G$ is said to be a subgroup of $G$ if, under the product of $G$, $H$ itself forms a group.
   
   (b) What does it mean that a subgroup $H$ of $G$ is normal?
   A subgroup $N$ of $G$ is said to be a normal subgroup of $G$ if for every $g \in G$ and $n \in N$, $gng^{-1} \in H$.
   Equivalently, $N$ is a normal subgroup of $G$ if and only if $gNg^{-1} = N$ for all $g \in G$.
   
   (c) What is the centralizer $C$ of a subgroup $H$ of $G$?
   If $H$ is a subgroup of $G$, then the centralizer $C(H)$ of $H$ is the set $\{x \in G \mid xh = hx \text{ for all } x \in H\}$
   
   (d) What is the commutator subgroup $B$ of a group $G$.
   The commutator subgroup of $G$ is the subgroup generated by the elements of the form $ghg^{-1}h^{-1}$.
   
   (e) If $\phi : G \to G'$, what does it mean that $\phi$ is a group homomorphism?
   A mapping $\phi : G \to G'$ is said to be a homomorphism if $\phi(gh) = \phi(g)\phi(h)$ for all $g$ and $h \in G$.

2. (10) Find the center of the dihedral group $D_{10}$.
   The center of a group consists of those elements that commute with every group element.

3. (10) Let $M$ and $N$ be normal subgroups of $G$. Show that $MN$ is a normal subgroup of $G$.

4. (10) Let $G$ be a group and $g \in G$. Set $\phi(x) = g^{-1}xg$. Show that $\phi$ is a homomorphism.

5. (10) Let $G$ be a group in which $(ab)^3 = a^3b^3$ for all $a$ and $b$ in $G$. Show that $H = \{x^3 \mid x \in G\}$ is a normal subgroup of $G$.

6. (10) Suppose that $H$ is a subgroup of $G$ such that whenever $Ha \neq Hb$, then $aH \neq bH$. Prove that $gHg^{-1} \subseteq H$ for all $g \in G$.

7. (20) If $G$ is an abelian group of order $o(G)$, and if $p$ is a prime number, such at $p^n \mid o(G)$, $p^{n+1} \not\mid o(G)$, then $G$ has a subgroup of order $p^n$. Prove it.