Applied Calculus (Math 215) – Midterm 2

Problem 1. [5 Points] State the Second Derivative Test for finding local extrema of a function $f(x)$.

Problem 2. [5 Points] State what it means that a function $f(x)$ is concave up on an interval I. Illustrate your definition with the picture of an example.

Problem 3. [10 Points] Solve the initial value problems:

(a) $f'(x) = 2x + \sec^2 x$ and $f(\pi/4) = 1$.

(b) $g'(x) = 3g(x)$ and $g(5) = 1$.

Problem 4. [20 Points] Cut a string of length 1 meter into two pieces. Use one piece to form a circle (the perimeter of a disk) and one as the perimeter of a square. How should the string be cut so that the combined area of the square and disk is maximal? What is the largest possible combined area? How should the string be cut, so that the combined area of the square and the disk is minimal? What is the smallest possible combined area?

Problem 5. [35 Points] Consider the function $f(x) = x^3 - 4x + 3$.

(a) Find the intercepts of the graph with the x and y axes.

(b) Compute the first and second derivative of f.

(c) Find the intervals on which the function is increasing, resp., decreasing.

(d) Find the local extrema of f.

(e) Find the intervals on which the function is concave up, resp., down.

(f) Find the inflection points of f.

(g) Calculate $f(2)$ and $f'(2)$. Use these values and approximation by differentials to get an approximate value of $f(2.2)$.

(h) What are the absolute extrema of the function on the interval $[-3, 2.2]$?

(i) Use all of your previous calculations to sketch the graph of the function.

Problem 6. [15 Points] Suppose $y(x)$ is a solution of the initial value problem:

$$y' = (x^2 - y)\tan y \quad \text{and} \quad y(2) = \pi/4.$$

(a) Find the tangent line to the graph of $y(x)$ at $(2, \pi/4)$.

(b) Find an approximate value for $y(2.1)$.

(c) Find y'' and decide whether $y(x)$ is concave up or down at $x = 2$.

Problem 7. [10 Points] Find a zero of the function $f(x) = \cos^2 x - \sin x$. Use $x_0 = \pi/4$ as a first guess. Find an improved guess x_1 using Newton's Method. Evaluate $f(x_1)$.