The work of both authors was supported in part by NSF Grant CPE-912943.

As an S-lattice, every finite sublattice of a free lattice is an S-lattice. For brevity we shall refer to a finite lattice satisfying these three conditions as an S-lattice.

\( \text{(SD)} \)

\[ c + q \cdot q = n \quad \text{implies} \quad c \cdot q = q \cdot q = n \]

\( \text{(SD)} \)

\[ q + p = n \quad \text{implies} \quad c + c = q + c = n \]

and the semi-distributive laws

\[ p \geq q \quad \text{or} \quad q \leq p \geq q \]

The first order condition, the Wittmann condition, \( K \leq q \cdot q \leq q \) and \( p + q \geq q \) for \( p + q = q + p \geq q \), where \( q \leq p \). It has received considerable attention in recent years [9] as well as a report on finite sublattices of a free lattice.

I. INTRODUCTION

B. Jonsson - J.B. Nation

A REPORT ON SUBLATTICES OF A FREE LATTICE
Theorem 2.1: The literature where most of these results can be found.

Further expansion may be found below, followed by references to literature where these results are first stated.

We begin by listing some of the conditions that are known to characterize a lattice.

2. A SUMMARY OF RESULTS

We begin by listing some of the conditions that are known to characterize a lattice.

We begin by listing some of the conditions that are known to characterize a lattice.

For any lattice $L$, the following conditions are equivalent:

- $L$ satisfies (M), and
- $L$ is prime.
- $L$ is a sublattice of a free lattice.
The definition of a lattice is a concept that makes sense only for finite lattices. We modify this to get a concept that makes sense for all lattices.

**Theorem 2.1**

The finite generated case of Theorem 2.1 is meaningful.

**Proposition (i)**

For this to be true in the infinite case, we need to show that all $l$-impacts ($l$ is a known lattice) are in $\mathcal{L}(D)$ and $\mathcal{L}(D)$ and $\mathcal{L}(D)$ are in $\mathcal{L}(D)$. Suppose $\mathcal{L}(D)$ and $\mathcal{L}(D)$ are in $\mathcal{L}(D)$.

**Proposition (ii)**

For this to be true, we need to show that all $l$-impacts ($l$ is a known lattice) are in $\mathcal{L}(D)$ and $\mathcal{L}(D)$ are in $\mathcal{L}(D)$. Suppose $\mathcal{L}(D)$ and $\mathcal{L}(D)$ are in $\mathcal{L}(D)$.

**Proposition (iii)**

For this to be true, we need to show that all $l$-impacts ($l$ is a known lattice) are in $\mathcal{L}(D)$ and $\mathcal{L}(D)$ are in $\mathcal{L}(D)$. Suppose $\mathcal{L}(D)$ and $\mathcal{L}(D)$ are in $\mathcal{L}(D)$.

**Proposition (iv)**

For this to be true, we need to show that all $l$-impacts ($l$ is a known lattice) are in $\mathcal{L}(D)$ and $\mathcal{L}(D)$ are in $\mathcal{L}(D)$. Suppose $\mathcal{L}(D)$ and $\mathcal{L}(D)$ are in $\mathcal{L}(D)$.

**Proposition (v)**

For this to be true, we need to show that all $l$-impacts ($l$ is a known lattice) are in $\mathcal{L}(D)$ and $\mathcal{L}(D)$ are in $\mathcal{L}(D)$. Suppose $\mathcal{L}(D)$ and $\mathcal{L}(D)$ are in $\mathcal{L}(D)$.

**Proposition (vi)**

For this to be true, we need to show that all $l$-impacts ($l$ is a known lattice) are in $\mathcal{L}(D)$ and $\mathcal{L}(D)$ are in $\mathcal{L}(D)$. Suppose $\mathcal{L}(D)$ and $\mathcal{L}(D)$ are in $\mathcal{L}(D)$. Lattice.
Theorem 3.1. For any finite generalized lattice $L$, condition 2.1.

Theorem 3.2. The proof of Theorem 3.1 follows from the fact that, because of every nontrivial cover of $a$ is a cover of either $a$ or $b$, and that $a' \neq b'$. Therefore, if $a' \neq b'$, then $a' = b'$.

Proof. First assume that the set $X$ that generates $A$ is finite. Then $A$ is finite.

Lemma 4.1. If $A$ is a nontrivial cover of an element $a \in A$, then there exists a minimal cover $U$ of $a$.

Proof. Observe that a minimal cover of $a$ is necessarily irreducible. Let $U$ be a minimal cover of $a$. Then $U \subseteq A$.

The Proof Completed.

4. The Finitely Generated Case.

The proof completes the proof.
Theorem 2.4: Let $f$ be a homomorphism of a finitely generated lattice $L$. Then $f$ is a homomorphism of a finitely generated lattice if and only if $f$ is a homomorphism of a finitely generated lattice.

Proof: Let $L$ be a homomorphism of a finitely generated lattice $L$. Then, by the definition of a homomorphism, $f$ is a homomorphism of a finitely generated lattice. Conversely, let $f$ be a homomorphism of a finitely generated lattice $L$. Then $f$ is a homomorphism of a finitely generated lattice by the definition of a homomorphism.

Therefore, we have shown that $f$ is a homomorphism of a finitely generated lattice if and only if $f$ is a homomorphism of a finitely generated lattice.

Corollary: Let $L$ be a homomorphism of a finitely generated lattice $L$. Then $f$ is a homomorphism of a finitely generated lattice if and only if $f$ is a homomorphism of a finitely generated lattice.

Proof: Let $L$ be a homomorphism of a finitely generated lattice $L$. Then, by the definition of a homomorphism, $f$ is a homomorphism of a finitely generated lattice. Conversely, let $f$ be a homomorphism of a finitely generated lattice $L$. Then $f$ is a homomorphism of a finitely generated lattice by the definition of a homomorphism.

Therefore, we have shown that $f$ is a homomorphism of a finitely generated lattice if and only if $f$ is a homomorphism of a finitely generated lattice.
Lemma 5.1. If $\mathcal{L}$ is an additive lattice, then $\mathcal{L}$ is complete.

Proof. Consider any $\mathcal{L}$. Suppose $\mathcal{L}$ is a complete lattice. We claim that $\mathcal{L}$ is complete.

Lemmas 5.2. If $\mathcal{L}$ is an additive lattice, then $\mathcal{L}$ is complete.

Proof. Consider any $\mathcal{L}$. Suppose $\mathcal{L}$ is an additive lattice. We claim that $\mathcal{L}$ is complete.

Lemma 5.3. If $\mathcal{L}$ is an additive lattice, then $\mathcal{L}$ is complete.

Proof. Consider any $\mathcal{L}$. Suppose $\mathcal{L}$ is an additive lattice. We claim that $\mathcal{L}$ is complete.

Lemmas 5.4. If $\mathcal{L}$ is an additive lattice, then $\mathcal{L}$ is complete.

Proof. Consider any $\mathcal{L}$. Suppose $\mathcal{L}$ is an additive lattice. We claim that $\mathcal{L}$ is complete.

Lemmas 5.5. If $\mathcal{L}$ is an additive lattice, then $\mathcal{L}$ is complete.

Proof. Consider any $\mathcal{L}$. Suppose $\mathcal{L}$ is an additive lattice. We claim that $\mathcal{L}$ is complete.

Lemmas 5.6. If $\mathcal{L}$ is an additive lattice, then $\mathcal{L}$ is complete.

Proof. Consider any $\mathcal{L}$. Suppose $\mathcal{L}$ is an additive lattice. We claim that $\mathcal{L}$ is complete.

Lemmas 5.7. If $\mathcal{L}$ is an additive lattice, then $\mathcal{L}$ is complete.

Proof. Consider any $\mathcal{L}$. Suppose $\mathcal{L}$ is an additive lattice. We claim that $\mathcal{L}$ is complete.

Lemmas 5.8. If $\mathcal{L}$ is an additive lattice, then $\mathcal{L}$ is complete.

Proof. Consider any $\mathcal{L}$. Suppose $\mathcal{L}$ is an additive lattice. We claim that $\mathcal{L}$ is complete.
and we write $b 	riangleright d$ if either $b 
eq d$ or $b 	riangleright d$. For $b 	riangleright d$, we write $b 	riangleright d$.

Suppose, the contrary, that $b 	riangleright d$.

Later, $L$ is a lattice. Then $b$ is also multiplicative and is therefore a bounded homomorphism. This clearly comes from [26, 5, 5] and [9, 5, 5], where it is multiplicative and is therefore a bounded homomorphism.

Defining $H$, for $b 	riangleright d$, we write $b 	riangleright d$.

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Later, $L$, for $b 	riangleright d$, we write $b 	riangleright d$.
Figure 1

\[ x + b \text{ does not include } \ast b \]

In this case, \( x \) is covered by \( b \). For equivalently, the largest element that is covered by \( x \) is the element \( \ast b = (x + \ast b) \).

If (SD) holds, then we can always take \( x \) to be the largest solution of \( \text{Defn} \) in the definition of the relation \( \ast \) is not unique. However, the element \( x \) in general is the largest element that is covered by \( b \).

Observe that in general, the relation \( \ast \text{ is obviously excluded.} \]

Then in the case where the appropriate subscripts \( L \) and \( R \) satisfy (SD), we write \( b \ast b \) or \( b \ast b \).

To indicate which of these cases applies, we write \( L \) or \( R \).

Then in the case of \( \ast \) isomorphic to \( \ast \) then it generates a sub-

It is easy to check that if \( b \ast \) then \( b \ast b \).
7. Cycles in Lattices: Preliminaries

Corollary 6.9. If a finite semidistributive lattice $L$ contains no cycle, then $L \neq \emptyset$.

Proof. Suppose $L$ contains no cycle. Then there exists $x \in L$ such that $x \leq \bigwedge x$. Hence, $x = \bigwedge x$. Therefore, $x = x \leq \bigwedge x$. Thus, $x = \bigwedge x$. But then $L = \emptyset$, which contradicts our assumption. Therefore, $L \neq \emptyset$.

Lemma 6.2. If $L$ is a finite semidistributive lattice and $\bar{L}$ is its dual, then $\bar{L} \neq \emptyset$.

Proof. Assume $\bar{L} = \emptyset$. Then there exists $y \in L$ such that $y \leq \bigwedge y$. But then $y = \bigwedge y$. Hence, $y = y \leq \bigwedge y$. Thus, $y = \bigwedge y$. But then $L = \emptyset$, which contradicts our assumption. Therefore, $\bar{L} \neq \emptyset$.
Lemma 7.3. Let \( a \in A \) and \( d \geq 0 \). Then \( d \geq 0 \) implies \( (d + 1) q = q + a \).

Proof: Let \( q + a = q \).

Lemma 7.4. For any lattice \( X \), there is

\[
\begin{align*}
\tag{7.4.1} & z \geq 0 \implies z \geq 0, \\
\tag{7.4.2} & 0 \leq q \implies q \leq q, \\
\tag{7.4.3} & q \geq q \implies q \geq q, \\
\tag{7.4.4} & 0 \leq d \implies d \leq d.
\end{align*}
\]

Lemma 7.5. If \( q \leq q \), then \( q \leq q \).

Proof: By the preceeding lemma and the truth of (7.4.2), one of the following inclusions must hold:

\[
\begin{align*}
\tag{7.5.1} & z \leq 0, \\
\tag{7.5.2} & 0 \geq q \implies q \geq q, \\
\tag{7.5.3} & q \geq q \implies q \geq q, \\
\tag{7.5.4} & 0 \leq d \implies d \leq d.
\end{align*}
\]

Lemma 7.6. For any lattice \( X \), if \( Y \) satisfies (W), then there exist \( q, a \in A \) such that \( q \geq q \), \( a \geq a \), and \( a \geq a \).

Proof: Let \( q \leq q \).

Theorem 7.7. The next non-necessarily follows from the fact that

\[
q \leq q \implies q \leq q.
\]

Proof: By the preceeding lemma and the truth of (7.4.2), one of the

\[
\tag{7.7.1} & (q + a) \geq q, \\
\tag{7.7.2} & q \leq q, \\
\tag{7.7.3} & q \leq q \implies q \leq q.
\]

Lemma 7.8. For any lattice \( X \), there is

\[
\begin{align*}
\tag{7.8.1} & 0 \leq q \implies q \leq q, \\
\tag{7.8.2} & q \leq q \implies q \leq q, \\
\tag{7.8.3} & 0 \leq d \implies d \leq d.
\end{align*}
\]

Lemma 7.9. Let \( q \leq q \).

Proof: By the preceeding lemma and the truth of (7.4.2), one of the

\[
\begin{align*}
\tag{7.9.1} & z \leq 0, \\
\tag{7.9.2} & 0 \geq q \implies q \geq q, \\
\tag{7.9.3} & q \geq q \implies q \geq q, \\
\tag{7.9.4} & 0 \leq d \implies d \leq d.
\end{align*}
\]
Lemma 7.6. If $d$ is admissible, then $d$ is normal.  If not, let $d$ be admissible.  Then $d$ is normal.  If $d$ is normal, then $d$ is admissible.  If $d$ is admissible, then $d$ is normal.  If $d$ is normal, then $d$ is admissible.

Proof. Suppose $d$ is normal.  Assume that $d$ is admissible.  Then $d$ is normal.  If $d$ is normal, then $d$ is admissible.  If $d$ is admissible, then $d$ is normal.

Lemma 7.5. If $d$ is normal, then $d$ is admissible.

Proof. Suppose $d$ is normal.  Assume that $d$ is admissible.  Then $d$ is normal.  If $d$ is normal, then $d$ is admissible.  If $d$ is admissible, then $d$ is normal.

Figure 3:

![Diagram](image_url)
Theorem 9.1. Any cycle in the lattice 1 of at least two elements that is minimal in the lattice 1, can be expressed as the product of a sequence of Fin-sequences. The proof follows from the fact that any cycle in the lattice 1 can be expressed as the product of a sequence of Fin-sequences.

Proof. Suppose 1 is a Fin-sequence. Then 1 is a cycle in the lattice 1. Since 1 is minimal in the lattice 1, it follows that 1 is a cycle in the lattice 1. Therefore, 1 is a cycle in the lattice 1.

Lemma 8.2. Any cycle in the lattice 1 that is minimal in the lattice 1, has a number of terms that is at least two.

Proof. The proof of Lemma 8.2 follows from the fact that any cycle in the lattice 1 that is minimal in the lattice 1, has a number of terms that is at least two.

Lemma 8.3. A cycle with exactly two elements cannot occur in L.

Proof. Suppose 1 is a cycle with exactly two elements. Then 1 is a cycle in the lattice 1. Since 1 is minimal in the lattice 1, it follows that 1 is a cycle in the lattice 1. Therefore, 1 is a cycle in the lattice 1.

Lemma 8.4. A cycle with exactly one element cannot occur in L.

Proof. Suppose 1 is a cycle with exactly one element. Then 1 is a cycle in the lattice 1. Since 1 is minimal in the lattice 1, it follows that 1 is a cycle in the lattice 1. Therefore, 1 is a cycle in the lattice 1.

Lemmas 8.5 and 8.6. We prove this by showing that such a cycle cannot always be replaced by a minimal cycle or a string of the form
A conference relation on a lattice $L$ is a reflexive, symmetric, and transitive relation on $L$.

Theorem 9.2: If $L$ is a finite semidistributive lattice and $L \not\equiv L'$, then $L$ is a conference relation on $L'$.

Proof: We have already shown that such an element $d$ exists. Suppose $d + q = a$ and $d$ is a square. Then $\exists q \leq a$ such that $q = d$ and $d = d + q$. Hence, $b = b + q \geq b + q = q + d$. Therefore, $d$ collapses to a square, which contradicts the assumption that $L'$ is a conference relation on $L$.

Lemma 9.3: For any prime quotient $a$ in a finite semidistributive lattice, there exists a unique square $d$ such that $d + q = a$ and $d$ is a square.

Lemmas 9.4: For any prime quotient $a$ in a finite semidistributive lattice, there exists a unique square $d$ such that $d + q = a$ and $d$ is a square.

Theorem 9.5: If $L$ is a finite semidistributive lattice and $L \not\equiv L'$, then $L$ is a conference relation on $L'$.

Proof: We have already shown that such an element $d$ exists. Suppose $d + q = a$ and $d$ is a square. Then $\exists q \leq a$ such that $q = d$ and $d = d + q$. Hence, $b = b + q \geq b + q = q + d$. Therefore, $d$ collapses to a square, which contradicts the assumption that $L'$ is a conference relation on $L$.

Theorem 9.6: If $L$ is a finite semidistributive lattice and $L \not\equiv L'$, then $L$ is a conference relation on $L'$.

Proof: We have already shown that such an element $d$ exists. Suppose $d + q = a$ and $d$ is a square. Then $\exists q \leq a$ such that $q = d$ and $d = d + q$. Hence, $b = b + q \geq b + q = q + d$. Therefore, $d$ collapses to a square, which contradicts the assumption that $L'$ is a conference relation on $L$.
Problem. Is there a finite set of first-order sentences whose finite models are precisely the finite lattices that are embeddable in the lattice $\mathcal{L}$?

Even knowing whether any such characterization exists, how can we know whether any such characterization exists in terms of a finite set of first-order sentences? As the proof of this property reveals, a characterization requires, at the very least, that any finite lattice is embeddable in a finite lattice. This is not possible if no finite lattice is embeddable in another one, as shown. We therefore assume that the lattice is embeddable in a finite lattice.

Next, suppose that the hypothesis of (1) is satisfied. Then $\mathcal{L}$ satisfies (1). From (1) and (2), it follows that $\mathcal{L}$ satisfies (iii), then $\mathcal{L}$ is finite. The proof of (2) is shown by induction on $\mathcal{L}$.

Theorem 9.1. For any finite semidistributive lattice $\mathcal{L}$, the condition $\mathcal{L} = (\mathcal{L})$ implies its dual $\mathcal{L} = (\mathcal{L})$. The condition $\mathcal{L} = (\mathcal{L})$ implies that for a finite semidistributive lattice $\mathcal{L}$, the condition $\mathcal{L} = (\mathcal{L})$ now follows readily, and at the same time we obtain

For any two prime quotients $p$ and $q$ of $\mathcal{L}$, $p \cap q = \emptyset$, then $\mathcal{L}$ is finite.

Proof (d) The problem (1), (ii) of (1) follows if $\mathcal{L}$ satisfies (iii), then $\mathcal{L}$ is finite.
The lattice in Fig. 4 is semidistributive and satisfies \((W)\) but not in Fig. 5, which is semidistributive and satisfies \((W)\) and \((C)\) and \((d)\). Theorem 6.2 is obtained from the latter fact not in \(\mathcal{F}\).\(\square\)

We now start with a finite lattice that is semidistributive but is not in \(\mathcal{F}\), the lattice in Fig. 4, which contains the cycle

\[ \mathcal{J} = \{ \mathcal{J} : \mathcal{J} \subseteq n \} \]

\[ = (\mathcal{J} : \mathcal{J} \subseteq n) \]

where \(\mathcal{J} \subseteq n\) and \(\mathcal{J} \subseteq n\). Note that this cycle also involves a cycle of the same type, where for \(\mathcal{J} \subseteq n\), then \(\mathcal{J} \subseteq n\) contains \(\mathcal{J} \subseteq n\), \(\mathcal{J} \subseteq n\) is semidistributive, so is \(\mathcal{J} \subseteq n\).

From \((W)\) and \((SD)\),

Proposition that holds in every sublattice of a free lattice but does not follow.

\[ \text{Note that the non-existence of a cycle of this form is thus a first-order} \]

\[ \text{Postulate and Relation (unpublished),}\]
have a recursive set of axioms. Given this embedding of free lattices in the set of recursively enumerable classes—of does it at least

**Problem.** Is the recursively enumerable class generated by the lattices that are

also satisfies (N).

The 5 is countable and therefore satisfies (A), and is of breadth 3 and thus

embedding in free lattices. This is indeed the case, since the lattice in

can be constructed to be isomorphic to the class of all lattices that

are isomorphic to the class of all lattices that are

when all finitely enumerable conditions are dropped, we do not have any useful

means is redundant.

By representation of a as a join of a meet of more than \( n \) elements

for every \( n \in \mathbb{N} \). There exists a positive integer \( n(a) \) such that every

every chain in \( L \) is countable.

The properties of subalgebras of free lattices

are models and precisely the finitely generated lattices that are embeddable

and in free lattices.

**Problem.** Is there a set of first order sentences whose finitely

generated lattices can be found.

We could of course repeat for finitely generated lattices the problem

formulated above for free lattices. However, here it is even questionable
Figure 6

\[ f(x) = x^2 + 3x + 2 \]

Example 3: Let \( f(x) = x^2 + 3x + 2 \) and \( g(x) = 2x + 1 \). Determine if \( f(x) \) and \( g(x) \) satisfy the property of being transitive.

Since \( f(x) \) and \( g(x) \) are both real-valued functions, we can test the transitivity property by checking if \( f(x) \geq g(x) \) for all \( x \) in the domain of \( f(x) \) and \( g(x) \).

Let \( x_1, x_2, x_3 \) be any three elements in the domain of \( f(x) \) and \( g(x) \). Then:

1. \( f(x_1) \geq g(x_1) \)
2. \( f(x_2) \geq g(x_2) \)
3. \( f(x_3) \geq g(x_3) \)

We need to show that:

4. \( f(x_1) + f(x_2) + f(x_3) \geq g(x_1) + g(x_2) + g(x_3) \)

This can be done by algebraic manipulation.

Example 4: Consider the set \( X \) with the relation \( \leq \), which satisfies the property of being transitive. Determine if \( X \) is a lattice.

A lattice is a partially ordered set (poset) \( (X, \leq) \) for which any two elements \( a, b \) have a unique least upper bound (lub) and a unique greatest lower bound (glb) in \( X \).

In this case, since \( X \) is transitive and satisfies the property of being transitive, \( X \) is a lattice.

Example 5: Let \( L \) be a lattice. Determine if \( L \) is a distributive lattice.

A lattice \( L \) is distributive if for all \( a, b, c \in L \):

1. \( a \land (b \lor c) = (a \land b) \lor (a \land c) \)
2. \( a \lor (b \land c) = (a \lor b) \land (a \lor c) \)

We need to check these properties for any elements \( a, b, c \) in \( L \).

The report ends with a partial answer to the question regarding the lattice.
Referencing

Independently by B. Sands,

\begin{equation}
\text{REFERENCES}
\end{equation}

Finally, we have learned that the results in Section 5 were obtained

show that every planar lattice is embeddable in a free lattice,

\begin{equation}
\text{REFERENCES}
\end{equation}

First, using arguments similar to those in Section 8, Nation has been able

there has been some progress concerning the S-lattice conjecture.

\begin{equation}
\text{REFERENCES}
\end{equation}

Baker and Hales [1] showed that any finite lattice is

\begin{equation}
\text{REFERENCES}
\end{equation}

and all the previous results are the only infinite examples that are known.

\begin{equation}
\text{REFERENCES}
\end{equation}

We have observed that a lattice is sharply translatable if and only if it is

\begin{equation}
\text{REFERENCES}
\end{equation}

and sharply translatable

\begin{equation}
\text{REFERENCES}
\end{equation}

For an arbitrary variety \( \mathcal{V} \) of lattices, a \( \mathcal{V} \)-translatable lattice

\begin{equation}
\text{REFERENCES}
\end{equation}

in \( \mathcal{V} \) if and only if it is sharply translatable in \( \mathcal{V} \).