MATH 100 - WORKSHEET SOLUTIONS

1. Worksheet 2A

To determine whether the equation \( ax = b \mod n \) has a solution, first find the greatest common divisor \( d \) of \( a \) and \( n \). If \( d \) is a divisor of \( b \), then the equation has \( d \) solutions in \( \mathbb{Z}_n \). If not, there are no solutions.

Find the solutions to these equations, if there are solutions.

(1) \( 2x = 3 \mod 4 \). No solution.
(2) \( 2x = 2 \mod 4 \). \( x = 1, 3 \mod 4 \)
(3) \( 2x = 5 \mod 10 \). No solution.
(4) \( 7x = 1 \mod 10 \). \( x = 3 \mod 10 \)
(5) \( 7x + 1 = 6 \mod 10 \). \( x = 5 \mod 10 \)
(6) \( 4x + 2 = 0 \mod 10 \). \( x = 2, 7 \mod 10 \)
(7) \( 6x = 3 \mod 9 \). \( x = 2, 5, 8 \mod 9 \)
(8) \( 6x + 1 = 0 \mod 9 \). No solution.
(9) \( 2x = 5 \mod 7 \). \( x = 3 \mod 7 \)
(10) \( 4x = 1 \mod 7 \). \( x = 2 \mod 7 \)
(11) \( 4x + 1 = 1 \mod 7 \). \( x = 0 \mod 7 \)
(12) \( 4x + 4 = 1 \mod 7 \). \( x = 1 \mod 7 \)
(13) \( 7x = 2 \mod 11 \). \( x = 5 \mod 11 \)
(14) \( 3x = 1 \mod 19 \). \( x = 13 \mod 19 \)

2. Worksheet 2B

Find the solutions to these equations, if there are solutions.

(1) \( 2x = 3 \mod 6 \). No solution.
(2) \( 2x = 2 \mod 6 \). \( x = 1, 4 \mod 6 \)
(3) \( 4x = 7 \mod 10 \). No solution.
(4) \( 7x = 1 \mod 10 \). \( x = 3 \mod 10 \)
(5) \( 5x + 1 = 6 \mod 10 \). \( x = 1, 3, 5, 7, 9 \mod 10 \)
(6) \( 3x + 4 = 0 \mod 10 \). \( x = 2 \mod 10 \)
(7) \( 6x = 0 \mod 12 \). \( x = 0, 2, 4, 6, 8, 10 \mod 12 \)
(8) \( 4x + 2 = 0 \mod 12 \). No solution.
(9) \( 4x = 5 \mod 7 \). \( x = 3 \mod 7 \)
(10) \( 3x = 1 \mod 7 \). \( x = 5 \mod 7 \)
(11) \( 2x + 3 = 3 \mod 7 \). \( x = 0 \mod 7 \)
(12) \( 4x + 4 = 0 \mod 7 \). \( x = 6 \mod 7 \)
(13) \( 7x = 2 \mod 13 \). \( x = 4 \mod 13 \)
(14) \( 4x + 14 = 0 \mod 15 \). \( x = 4 \mod 15 \)

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