(1) Find two nonnegative real numbers that add up to 50 and whose product is as large as possible.

(2) A coffee farmer in Kona wants to fence off a rectangular coffee field with 2500 feet of fence and with one side of the field bordering a straight river. What are the dimensions of the field that yield the maximum area.

(3) Let $Y(N)$ be the yield of an agricultural crop as a function of the nitrogen level $N$ in the soil. A model that is used for this is

$$Y(N) = \frac{N}{1 + N^2} \text{ for } N \geq 0$$

(where $N$ is measured in appropriate units). Find the nitrogen level that maximizes yield.

(4) Soda cans hold about 12 ounces of liquid, and 12 fluid ounces is approximately 355 mL (or 355 cm$^3$). Find the dimensions of a can which will minimize the cost of metal.

(5) (Problem taken from “How to Ace Calculus”) You are in a row boat on Lake Erie, 2 miles from a straight shoreline, taking your potential in-laws for a boat ride. Six miles down the shoreline from the nearest point on shore is an outhouse. You suddenly feel the need for its use. Also, the shore line is populated with lots of houses, all owned by people who know your parents. If you can row at 2 mph and run at 6 mph, for what point along the shoreline should you aim in order to minimize the amount of time it will take you to get to the outhouse?

(6) (Problem taken from “How to Ace Calculus”) You have just invented a new peanut butter guacamole dip, and you open a stand in front of the student union to sell this goop by the jar. Somehow a rumor gets started, certainly not traceable back to you, that it is an aphrodisiac, and sales take off. At a price of $1.00 a jar, you sell 500 jars a day. For each nickel that you increase the price, you sell two fewer jars. Assuming that your fixed cost per day is $200 (protection money), and the cost per jar to you is 50 cents, determine the price for which you should sell your dip in order to maximize your profit.