

Name: _____

Student ID: _____

Instructions. Write your answers in the spaces provided on this exam. You may NOT use a calculator. This exam is closed book and closed notes.

(1) Find the limits if they exist. If a limit does not exist, explain why.

(a) $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - 4}$

(b) $\lim_{x \rightarrow 2} \frac{\sqrt{x} - \sqrt{2}}{x - 2}$

(c) $\lim_{x \rightarrow 0} x \sin \frac{1}{x}$

(d) $\lim_{x \rightarrow 0} \frac{\sin x}{x^2 + 2x}$

(2) Differentiate. You need not simplify complicated expressions.

(a) $f(x) = 5x^4 + 4.2 + \frac{1}{x^2}$

(b) $g(x) = x^{45} \tan x$

(c) $h(t) = 7^t + t^7$

(d) $y = \frac{x^2}{x^4 + x + 1}$

(e) $m(t) = \sqrt{t + \cos t}$

(f) $n(x) = \ln(x^2 + \pi x + 4)$

(g) $q(t) = \arctan(t + 1)$

(h) $r(\theta) = \sin^3 \theta$

(3) Find the tangent line to the curve $y = 2^x$ at $x = 0$.

(4) Find $\frac{dy}{dx}$ for the curve $x^2y + y^2 = 4$. At what points is the tangent line horizontal?

(5) Consider the function

$$h(x) = \begin{cases} (1+x)^{\frac{1}{x}} & \text{if } x \neq 0 \\ e & \text{if } x = 0 \end{cases}$$

What limit would you take to find $h'(0)$?

(6) Complete the definition: $\lim_{x \rightarrow a} f(x) = L$ if

State an important theorem about differentiable functions.

(7) Prove that $\lim_{x \rightarrow 2} 5x = 10$, OR for extra credit, prove that $\lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0$.