Let $M$ be a symmetric $n \times n$ matrix, and $Q(x) = x^* M x$ its associated quadratic form. We want methods to tell whether $M$ is positive definite, that is, whether $Q(x) > 0$ for every $x \neq 0$.

For simplicity, we will assume that $M$ is invertible, so that zero is not an eigenvalue.

1. **Necessary conditions**

   1. In order for $M$ to be positive definite, the diagonal elements must all be positive. Likewise, for $M$ to be negative definite, the diagonal elements must all be negative.

   2. The determinant of $M$ is the product of its eigenvalues. If the size $n$ is even and $\det(M) < 0$, then the eigenvalues must have different signs, so $M$ is neither positive nor negative definite. If $n$ is odd and the determinant is negative, the eigenvalues are not all positive. If $n$ is odd and the determinant is positive, the eigenvalues are not all negative.

2. **Necessary and sufficient conditions**

   3. It is always possible, and sometimes easy, to write $Q(x)$ as a sum or difference of squares, which makes positive or negative definiteness transparent.

   Let $p(\lambda) = \det(M - \lambda I)$ denote the characteristic polynomial of $M$. If there is a computer or graphing calculator handy we can approximate the eigenvalues - the roots of $p(\lambda)$ - numerically easily enough. But there is an easier way to determine positive definiteness.

   4. Because there are no non-real (complex) roots of $p(\lambda)$, Descartes’ Rule of Signs says that the number of positive roots of $p(\lambda)$ is the number of changes of sign in its coefficients. Thus $M$ is positive definite if and only if $p(\lambda)$ has $n$ changes of sign in its coefficients. Likewise, $M$ is negative definite if and only if $p(\lambda)$ has no changes of sign in its coefficients.

   5. Here is a method that avoids computing $p(\lambda)$.

      If the diagonal of $M$ is all positive, then $M$ is positive definite if and only if the determinants of all the upper left-hand corners are positive.

      If the diagonal of $M$ is all negative, then $M$ is negative definite if and only if the determinants of all the upper left-hand corners are alternate in sign.