(1) Find the mass of the tetrahedron bounded by the planes \( x = 0, \ y = 0, \ z = 0, \) and \( x+y+z = 1 \) if the density is given by \( \delta(x, y, z) = 1-z. \)

\[
\int_0^1 \int_0^{1-x} \int_0^{1-x-y} 1-z \, dz \, dy \, dx = \frac{1}{8}
\]

(2) Find the centroid of the region between the xy-plane and \( z = 1-x^2-y^2. \)

The volume is

\[
\int_0^{2\pi} \int_0^1 \int_0^{1-r^2} r \, dz \, dr \, d\theta = \frac{\pi}{2}
\]

while

\[
V \, \mathbf{\bar{z}} = \int_0^{2\pi} \int_0^1 \int_0^{1-r^2} z \cdot r \, dz \, dr \, d\theta = \frac{\pi}{6}
\]

so \( \mathbf{\bar{z}} = \frac{\pi/6}{\pi/2} = \frac{1}{3}. \) By symmetry \( \mathbf{\bar{x}} = \mathbf{\bar{y}} = 0. \)

(3) Find the mass of the earth if the density is given by \( \rho(x, y, z) = \frac{K}{\rho}. \)

(4) Find the length of the curve \( y = \sqrt{x} \) from \( x = 0 \) to \( x = 1. \)

With the parameterization \( x(t) = t^2, \ y(t) = t, \ 0 \leq t \leq 1 \) you get

\[
s = \int_0^1 \sqrt{1+4t^2} \, dt = \frac{\sqrt{5}}{2} + \frac{1}{4} \ln(2 + \sqrt{5})
\]

(5) Find the line integral \( \int_L (2x + 2y) \, dx + (2x + 9y^2) \, dy \) along the line from \( (0,1) \) to \( (2,4) \) both directly and by using the fundamental theorem.

Directly, use the parameterization \( x(t) = 2t, \ y(t) = 1+3t, \ 0 \leq t \leq 1. \) For the fundamental theorem, use the fact that \( (2x + 2y)i + (2x + 9y^2)j = \nabla(x^2 + 2xy + 3y^3) \) and \( \int_{A \rightarrow B} \nabla \phi \cdot dr = \phi(B) - \phi(A). \) Either way, the answer is 209.

(6) Find the line integral \( \int_L (2x \cos y + 3) \, dx + (x - x^2 \sin y) \, dy \) counterclockwise around the ellipse \( \frac{x^2}{2} + \frac{y^2}{5} = 1 \) both directly and by Green’s Theorem.

With Green’s Theorem, the integral is \( \int \int 1 \, dA = \pi \sqrt{10}. \) It seems nigh impossible to do this integral directly, but perhaps that shows the value of Green’s Theorem!

(7) Find \( \phi \) such that \( f = \nabla \phi: \ f(x,y) = (x + \sin y)i + (x \cos y + 5)j. \)

\( \phi = \frac{x^2}{2} + x \sin y + 5y \)

(8) Find \( \phi \) such that \( f = \nabla \phi: \ f(x, y, z) = (7yz^2 + 1)i + (7xz^2 + y)j + (14xyz + \cos z)k. \)

\( \phi = 7xyz^2 + x + \frac{y^2}{2} + \sin z \)

(9) How much work is required to stretch a spring with spring constant \( k \) from equilibrium to \( L \) units beyond equilibrium?

Measuring the extension \( x \) on an axis with the equilibrium at \( x = 0, \) the spring force is \( F = -kx = -\frac{dV}{dx}. \) So \( V = \frac{1}{2}kx^2 \) and the work done in the problem, which is the change in potential energy for a conservative force like this, is \( W = \frac{1}{2}kL^2. \)