A. Calculate the line integral of the vector field f along the path described.

(1) $f(x, y) = (x^2 - 2xy)\mathbf{i} + (y^2 - 2xy)\mathbf{j}$ from $(-1, 1)$ to $(1, 1)$ along the parabola $y = x^2$.

(2) $f(x, y, z) = (y^2 - z^2)\mathbf{i} + 2yz\mathbf{j} - x^2\mathbf{k}$ along the path $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ for $0 \le t \le 1$.

(3) $f(x, y) = (x + y)\mathbf{i} + (x - y)\mathbf{j}$ once around the ellipse $4x^2 + 9y^2 = 36$ in a counterclockwise direction.

(4) $f(x, y, z) = x\mathbf{i} + y\mathbf{j} + (xz - y)\mathbf{k}$ from $(0, 0, 0)$ to $(1, 2, 4)$ along a line segment.

(5) $f(x, y, z) = x\mathbf{i} + y\mathbf{j} + (xz - y)\mathbf{k}$ from $(0, 0, 0)$ to $(1, 2, 4)$ along the path $\mathbf{r}(t) = t^2\mathbf{i} + 2t\mathbf{j} + 4t^3\mathbf{k}$ for $0 \le t \le 1$.

B. In each case, determine whether the vector field f is the gradient of a scalar field. If so, find φ such that $f = \nabla\varphi$.

(1) $f(x, y) = x\mathbf{i} + y\mathbf{j}$.

(2) $f(x, y) = 3x^2y\mathbf{i} + x^3\mathbf{j}$.

(3) $f(x, y) = (2xe^y + y)\mathbf{i} + (x^2e^y + x - 2y)\mathbf{j}$.

(4) $f(x, y, z) = (x + z)\mathbf{i} - (y + z)\mathbf{j} + (x - y)\mathbf{k}$.

(5) $f(x, y, z) = 2xy^3\mathbf{i} + x^2z^3\mathbf{j} + 3x^2yz^2\mathbf{k}$.