Name:______________________________ Student ID:____________________

(1) List ten equivalents to the statement $\mathbf{M}$ *is nonsingular.*

(2) Find a basis for $\mathbf{W} = \{\mathbf{x} \in \mathbb{R}^4 : 2x_1 + 2x_2 - x_3 - 4x_4 = 0\}$.

(3) Solve $\mathbf{Ax} = \mathbf{b}$, and write the solution in vector form.

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 0 & -2 & -2 \\ 3 & -1 & -6 & -6 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} -1 \\ 4 \\ 9 \end{bmatrix}$$

(4) For $\mathbf{A}$ and $\mathbf{b}$ from the preceding problem, find $\|\mathbf{b}\|_1$, $\|\mathbf{A}\|_1$, $\|\mathbf{b}\|_\infty$, $\|\mathbf{A}\|_\infty$.

(5) Diagonalize the matrix. (Be careful with signs.)

$$\mathbf{B} = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

(6) Prove that if λ is an eigenvalue of $\mathbf{M}$, then $|\lambda| \leq \|\mathbf{M}\|$ (for any matrix norm).

(7) Prove that if the square matrix $\mathbf{P}$ satisfies $\mathbf{P}^2 = \mathbf{P}$, then the eigenvalues of $\mathbf{P}$ are 0, or 1, or both. Give an example where both occur.

(8) Give an example of a matrix $\mathbf{N}$ such that $N^4 = 0$ but $N^3 \neq 0$.

(9) Suppose $\mathbf{x}$ is an eigenvector for $\mathbf{M}$ corresponding to α, and $\mathbf{y}$ is an eigenvector for $\mathbf{M}^T$ corresponding to $\beta \neq \alpha$. What can you conclude?