II. For each of the following groupoids, determine the following.

(a) Is the algebra commutative?
(b) Is the algebra associative?
(c) Does it have an identity element? (If so, which?)
(d) Does it have a zero element? (If so, which?)
(e) Does it have idempotent elements? (If so, which?)
(f) Does it have the property that for every pair \(a, b\) the equation \(ax = b\) has a solution?

(1) \(\langle \mathbb{Z}_3, + \rangle\)
(2) \(\langle \mathbb{Z}_4, + \rangle\)
(3) \(\langle \mathbb{Z}_3, \times \rangle\)
(4) \(\langle \mathbb{Z}_4, \times \rangle\)
(5) The set \(\{0, 1, 2, 3, 4\}\) with the multiplication \(x \circ y = x\).
(6) The natural numbers \(\mathbb{N}\) with the operation \(x \wedge y\) being the minimum of \(x\) and \(y\).
(7) Jupiter
(8) Saturn
(9) Uranus
(10) Neptune

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\begin{array}{c|ccc}
* & R & P & S \\
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R & R & P & R \\
P & P & P & S \\
S & R & S & S \\
\end{array}
\]

Table 1. Jupiter

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P & P & R & S \\
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\end{array}
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Table 2. Saturn
Table 3. *Uranus*

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Table 4. *Neptune*

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