AGNOSTIC INSURANCE TASKS AND THEIR RELATION TO COMPRESSION

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We consider the problem of predicting finite upper bounds on unseen samples of an unknown distribution p over the set of natural numbers, N, using only observations generated i.i.d. from it. While p is unknown, it belongs to a known collection P of possible models. Each sample from p here represents the total loss suffered by the insured at a particular time step. The upper bound plays the role of the total built up reserves of an insurer, including past premiums after paying out past losses, as well as the premium charged at that time step based on estimate of future losses. We allow the insurer to observe losses for any finitely long time before stepping in. But the insurer cannot quit the game once entered, and the insurer must enter with probability 1.

If the loss is unbounded, is it possible for the insurer to set premiums so that the probability of bankruptcy, over even an infinitely long time, can be made arbitrarily small? Equivalently, when is P insurable? We present a condition that is both necessary and sufficient for any class P of distributions to be insurable.

The connections of insurability with compression turn out to be subtle. On the one hand, strong universal compression of P, which implies uniform bounds on codelengths over the entire class P, implies insurability as expected. However, given how we define insurability, we are more interested in parallels with other pointwise convergence properties—weak universal compression. Perhaps surprisingly, we show that neither weak compression nor insurability implies the other.