1. Let M be a left R-module. Show that the following conditions are equivalent:

(1) Every left invertible φ in $\operatorname{End}_R M$ is also right invertible.
(2) Every right invertible endomorphism is also left invertible.
(3) No proper summand of M is isomorphic to M.

2. **a)** Prove that a module with finite length over any ring is finitely generated.
b) Prove that if R is an artinian ring without zero divisors then R is a skew field.
c) Prove that if p is a prime ideal in a commutative ring R and R/p is an artinian R-module then p is maximal.
d) Prove that if M is a non-trivial module with finite length over a commutative ring R, then $\operatorname{Ass} M$ is not empty and consists of maximal ideals.

3. Let $M = K \oplus N$ be a direct sum of R-modules.
a) Prove that if L is a submodule of M such that $N \subseteq L$, then $L = N \oplus (L \cap K)$.
b) A submodule H of M is called **fully invariant** if $\varphi(H) \subseteq H$ for all $\varphi \in \operatorname{End}_R M$. Prove that if H is a fully invariant submodule of M then $H = (H \cap K) \oplus (H \cap N)$.
c) Prove that if H is a direct summand of M (i. e. $M = H \oplus H'$ for some H') and H is fully invariant in M, then $H \cap N$ is a direct summand of N.

4. Let L be a minimal (non-trivial) left ideal in a ring R and suppose that at least one maximal left ideal of R does not contain L.
a) Prove that L is a direct summand of the left R-module R.
b) Prove that L is generated by an idempotent (i. e. by an element e such that $e^2 = e$).

5. Let L be a minimal (non-trivial) left ideal in a ring R and suppose that there exists $x \in L$ such that $Lx \neq 0$. Prove that L is generated by an idempotent.
(HINT: Prove that the map $\varphi \in \operatorname{End}_R L$ given by $\ell \mapsto \ell x$ is an automorphism of L.)