MATH 612 **Due Wednesday March 17, 1993**

1. **a)** Prove that a direct product of a family of injective modules is injective.
b) Prove that if R is noetherian then a direct sum of an arbitrary family of injective modules is injective.

2. Let E be an injective module. Prove that the following conditions are equivalent:

(1) E is indecomposable.
(2) E is the injective envelope of each of its non-trivial submodules.
(3) Every two non-trivial submodules of E intersect non-trivially.

3. Prove that if E is an artinian indecomposable injective module then E contains a unique simple submodule S and that E is the injective envelope of S.

4. Prove that if E is an indecomposable injective module over a commutative noetherian ring R, then $\operatorname{Ass} E = \{\mathfrak{p}\}$ for some prime ideal $\mathfrak{p}$, E is isomorphic to the injective envelope of $R/\mathfrak{p}$, and $E \approx E_{\mathfrak{p}}$.

5. Let E be an indecomposable injective module. Prove that every monic endomorphism of E is in fact an automorphism and that $\operatorname{End}_R E$ is a local ring.