1. **Give a reason** why each series converges or diverges:

a) $\sum \frac{k^3+1}{2k^3+3k+5}$

b) $\sum \frac{k!}{k^k}$

c) $\sum \frac{k^3}{2^k}$

d) $\sum \frac{(-1)^k}{3k+2}$

e) $\sum \frac{\ln k}{k}$

f) $\sum \frac{3k+2}{k^3+4k+7}$

g) $\sum \frac{k^2}{k^3-4k+1}$

h) $\sum \frac{(-1)^k}{k}$

2. Find the Taylor series expansion for each function as indicated and give the radius of convergence. **Answers should be in summation notation.**

a) e^x in powers of $x-3$.

b) $\frac{1}{(1-x)^2}$ in powers of x.

c) $\tan^{-1} x^2$ in powers of x.

d) $\ln(x+5)$ in powers of $x+2$.

e) $\ln(1+x^3)$ in powers of x.

f) $\tan^{-1}\frac{x}{2}$ in powers of x.

g) $\sqrt{2+x}$ in powers of x.

h) $(1-3x)^{-2}$ in powers of x.

i) $\frac{1}{x-3}$ in powers of x.

j) $x^2\cos x^3$ in powers of x

k) xe^{2x^2} in powers of x

l) $\frac{1}{5-3x}$ in powers of x

m) $\frac{1}{x}$ in powers of $x-3$

n) e^{3x} in powers of $x-2$

o) e^{2x} in powers of $x-5$.

p) $\tan^{-1} 3x$ in powers of x

q) $\sqrt{1+3x}$ in powers of x

r) $\cos x$ in powers of $x-\frac{\pi}{2}$

s) $\ln x$ in powers of $x-1$

t) $x^2\sin x$ in powers of x

u) e^{-2x} in powers of $x+1$

v) $\sqrt{x}$ in powers of $x-2$

w) $x\ln\left(\frac{1+x^2}{1-x^2}\right)$ in powers of x

3. Find the radius of convergence for each series.

a) $\sum \frac{3^{2k}}{k} x^k$

b) $\sum \frac{k^2 x^k}{3^k}$

c) $\sum (-1)^k \frac{2^k x^k}{k!}$

d) $\sum_{k=2}^{\infty} \frac{4^k x^k}{(k-2)!}$

e) $\sum (-1)^k \frac{k^2}{k!} x^k$

f) $\sum_{k=0}^{\infty} \frac{k(x-2)^k}{5k^3+2}$

4. Sum (i. e. evaluate) each series.

a) $\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}$

b) $\sum_{k=0}^{\infty} \frac{(-1)^k (x-3)^{2k+1}}{2k+1}$

c) $\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$

d) $\sum_{0}^{\infty} (-1)^k \left(\frac{3}{4}\right)^{k+3}$

e) $\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k)!}$

f) $\sum_{k=0}^{\infty} 5x^{3k+2}$

g) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$

h) $\sum_{k=0}^{\infty} 2^k x^{k+5}$

i) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k\, 2^k}$

j) $\sum_{k=0}^{\infty} \frac{x^k}{3^{k+2}}$

k) $\sum_{1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1}\right)$

l) $\sum_{1}^{\infty} (-1)^{k+1} \frac{x^{3k}}{k}$

m) $\sum_{0}^{\infty} \frac{1}{k!}$ (NOTE: $0! = 1$)

n) $\sum_{0}^{\infty} (-1)^k \frac{x^k}{(2k)!}$

o) $\sum_{k=0}^{\infty} \frac{2^{k+1}}{3^k}$

p) $\sum_{k=1}^{\infty} \left(\frac{1}{k^2} - \frac{1}{(k+1)^2}\right)$

q) $\sum_{k=0}^{\infty} \frac{x^k}{6^k k!}$

r) $\sum_{1}^{\infty} \frac{1}{k(k+1)(k+2)}$

s) $\sum_{1}^{\infty} \frac{x^k}{(k-1)!}$

t) $\sum_{1}^{\infty} \frac{k^2}{k!}$