MATH 420 **First Mid-term** **NO CALCULATORS** Friday October 9, 1987

(15) **1.** **a)** State Wilson's Theorem.

b) Illustrate the idea of the proof of Wilson's Theorem by using the example $p = 13$.

(20) **2.** **a)** Find the smallest integer n such that $n \geq 8$ and $n \equiv 7 \pmod{4}$, $n \equiv 7 \pmod{5}$, and $n \equiv 7 \pmod{13}$.

b) Given that $12x \equiv 1 \pmod{59}$ has $x = 5$ as a solution, solve each of the following:

i) $12x \equiv 8 \pmod{59}$

ii) $12x \equiv 13 \pmod{59}$

iii) $12x \equiv 35 \pmod{59}$.

(15) **3.** Let a and b be positive integers. Prove that there exist x and y with $a = (x, y)$, $b = [x, y]$ if and only if $a | b$.

(25) **4.** **a)** Prove that the Diophantine equation $ax + by = c$ has a solution if and only if the congruence $ax \equiv c \pmod{b}$ has a solution.

b) Let $g = (a, b)$. By Theorem 1.4, $ax + by = g$ has a solution. **Use this** to prove that $ax + by = c$ has a solution if and only if $g | c$. (DO NOT use theorems about congruences.)

(25) **5.** **a)** Prove that for any n, $n^9 - n$ is divisible by 10.

b) Prove that $n^9 - n$ is divisible by 20 if and only if n does **not** have the form $4k + 2$.