

Periodic Decimal Expansions — Part I

E. L. Lady

MATH 111 UNIT IV SET 2.1

In this set we will look at repeating decimals from some new angles, seeing many new things about them.

1. What is the quick way to **multiply** a number in decimal form by a power of ten? Apply your rule by multiplying the following numbers by 1000:

3.872 .04315 731.69 $\overline{.142857}$ $.357\overline{1}$.

What is the quick way to **divide** numbers in decimal form by powers of 10? Apply your rule by dividing the following numbers by 100:

135.48 96.7 1834 974.58 $14.\overline{285714}$ $17.\overline{09}$

2. Divide 1000 by 37. What is the remainder?

Divide 999 by 37. What is the remainder?

Calculate the decimal expansion of $1/37$.

Divide 100,000 by 41. What is the remainder? Divide 99,999 by 41. What is the remainder? Calculate the decimal expansion of $1/41$.

What did you notice?

Can you find a number like 9, 99, 999, 9999, etc. that is divisible by 15? How can you be sure? How is the decimal expansion for $1/15$ different from the expansions of $1/37$ and $1/41$?

3. It will be convenient for us to classify non-terminating decimal expansions as either *immediate-repeating* or *delayed-repeating*. You can infer the meaning of these terms by working out the decimal expansion of the following examples:

Some fractions with immediate-repeating expansions:

$$\frac{2}{3} \quad \frac{8}{1} \quad \frac{5}{9} \quad \frac{2}{7} \quad \frac{16}{11} \quad \frac{10}{27}.$$

Some fractions with delayed-repeating expansions:

$$\frac{1}{15} \quad \frac{1}{6} \quad \frac{1}{35} \quad \frac{3}{55} \quad \frac{5}{18} \quad \frac{1}{12} \quad \frac{5}{54}.$$

Based on these examples, define the terms “immediate-repeating decimal” and “delayed-repeating decimal.”

Recall the method used in Set 2 for changing repeating decimals to fractions. When using this method, how does dealing with an immediate-repeating decimal (for instance $2.\overline{57}$) differ from dealing with a delayed-repeating decimal (for instance $16.8\overline{97}$)?

Make a guess as to what sort of fractions have delayed-repeating decimal expansions.

4. You will now learn a new method for converting immediate-repeating decimal to a fraction. I hope you will find it easier than the method you learned in Set 2. First, a few questions to put you in the right frame of mind.

What do you notice about the decimal expansions of $\frac{1}{37}$ and $\frac{1}{27}$? of $\frac{1}{271}$ and $\frac{1}{369}$?
of $\frac{1}{9}$ and $\frac{1}{11}$? of $\frac{1}{101}$ and $\frac{1}{99}$?

Write $1/27$ and $1/37$ as fractions with 999 as the denominator.

Write $1/271$ and $1/369$ as fractions with 99999 as the denominator.

Write $1/9$ and $1/11$ as fractions with 99 as the denominator.

Given that $1/909 = \overline{.0011}$, what fractions are represented by the following decimals?

$$\overline{.0022} \quad \overline{.0033} \quad \overline{.0044} \quad \overline{.0099}.$$

Calculate (**by hand**, please) the decimal expansions of $1/9$, $1/99$, $1/999$, $1/9999$.
Without calculating, what would be the decimal expansion of $1/99,999,999$?

Given that $\frac{1}{813} = \overline{.00123}$. Using the decimal expansion of $\frac{1}{99999}$, quickly calculate the decimal expansion of $\frac{123}{99999}$. What do you conclude about the fractions $\frac{1}{813}$ and $\frac{123}{99999}$?

Based on the calculations you have done, describe a new method for converting immediate-repeating decimals to fractions. Apply your method to the following examples:

$$\overline{.7} \quad 5.\overline{04} \quad 1.\overline{054} \quad \overline{.0198} \quad \overline{.00369} \quad 2.\overline{02439}.$$

(HINT: When multiplying integers by a number like 9999, notice that $9999 = 10000 - 1$ and use the distributive law for multiplication over subtraction. For instance, $7 \times 9999 = 70000 - 7$.)

Convert $\overline{.0198}$ and $5.\overline{04}$ to fractions using the old method from Set 2. In what ways is the new method like the old method?

What happens when you try and use the new method on a delayed-repeating decimal such as $\overline{.07}$, $\overline{.504}$, $\overline{.01054}$, $\overline{.000198}$, $\overline{.202439}$?

Can you think of a way of modifying the new method to deal with delayed-repeating decimals?

5. If a/b is a fraction in lowest terms, what must be true of b in order that a/b can be written as a fraction with a denominator of 9? of 99? of 999?

Notice that $9 = 10 - 1$, $99 = 100 - 1$, $999 = 10^3 - 1$, $9999 = 10^4 - 1$, etc.

EXPLAIN WHY: If a/b is in lowest terms and has an immediate-repeating decimal expansion of length r , then b must divide $10^r - 1$.