The Lazy Man Helps His Neighbor

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The Lazy Man's neighbor Mr. Tinker stopped by one evening and said he'd been working on some algebra problems. "My boy's homework's got these polynomials to work out, so I've been doing them. First you have to understand the way they talk. I'll tell you an example:

'Let $p(x) = 2x^5 + 9x^4 - 7x^3 + 5x^2 - 19x + 2$. Find $p(4)$.'

So finally I see that what they mean is, you should take this $2x^5 + \cdots$ gadget and substitute $x = 4$. I don't see why they can't just say things in simple words."

"I thought it was pretty clear," the Lazy Man said.

"Maybe to you it was clear, not to me. I notice you've got the same funny way of thinking that these books do. Anyway, it turns out that figuring these polynomials is a lot of work. That one I gave you, for instance. First you've got to find 4^5 and then multiply by 2, then find 4^4 and then multiply by 9, and —"

"You can do it easier than that," the Lazy Man said. "But I don't understand why you're doing your son's homework."

"The trouble is, the boy's on the team this year so he doesn't really have the time. But I figure somebody's better do it or they'll never let him into medical school. Anyway, if you've got a better way I'd like to know about it."

"Well, look. You need to raise 4 to the 5th power. So you start multiplying over and over by 4 and get 4   16   64   256   1024. But now you've not only found $4^5 = 1024$, but along the way've found $4^2 = 16$, $4^3 = 64$, and $4^4 = 256$. It's not really all that hard."

"Sure, I could see that," Mr. Tinker said. "But I didn't tell you the whole story. These are not just plain numbers like 4, they're full of decimals and things, so I wanted to do them on the calculator. But it's just a cheap one I got free at the bank for depositing $5,000, and it's only got one memory. So I can't save all those powers until I need them, and it's a lot of work getting all those numbers written down without getting any of them wrong."

"I guess I'd better teach you the most efficient way then," the Lazy Man said. First of all, think about rewriting your polynomial

$$2x^5 + 9x^4 - 7x^3 + 5x^2 - 19x + 2$$

as

$$((((2x + 9)x - 7)x + 5)x - 19)x + 2\,.$$

"Wait a minute. You just lost me there."

"Just use the distributive law, start inside the parentheses and work your way out. I'll do it in steps for $x = 4$."

$$\begin{aligned}
\mathbf{2} &\xrightarrow{\times 4} \mathbf{8} = 2x \\
\mathbf{8} + 9 = 17 &\xrightarrow{\times 4} \mathbf{68} = ((2x+9)x = 2x^2 + 9x \\
\mathbf{68} - 7 = 61 &\xrightarrow{\times 4} \mathbf{244} = ((2x+9)x - 7)x = 2x^3 + 9x^2 - 7x \\
\mathbf{244} + 5 = 249 &\xrightarrow{\times 4} \mathbf{996} = (((2x+9)x - 7)x + 5)x = 2x^4 + 9x^3 - 7x^2 + 5x \\
\mathbf{996} - 19 = 977 &\xrightarrow{\times 4} \mathbf{3908} = ((((2x+9)x) - 7)x) + 5)x - 19)x = 2x^5 + 9x^4 - 7x^3 + 5x^2 - 19x \\
\mathbf{3908} + 2 = \mathbf{3910} &= 2x^5 + 9x^4 - 7x^3 + 5x^2 - 19x + 2 \qquad \text{for } x = 4.
\end{aligned}$$

"That looks pretty complicated. I couldn't do it on the calculator. I'd need a dozen memories."

"No you wouldn't. First of all, the formulas on the right are only for explanation, so you don't need to write them down. Also, the column of numbers on the right, just after the arrows, is the same as the column of the left, so you don't need to write those down either. Once you get the rhythm of it, you don't even need one memory. Watch."

$$\begin{aligned}
2 \xrightarrow{\times 4} 8 \xrightarrow{+9} 17 \xrightarrow{\times 4} 68 \xrightarrow{-7} 61 \xrightarrow{\times 4} 244 \xrightarrow{+5} 249 \\
\xrightarrow{\times 4} 996 \xrightarrow{-19} 977 \xrightarrow{\times 4} 3908 \xrightarrow{+2} 3910\,.
\end{aligned}$$

"Finally, the calculation can be organized by writing the columns shown earlier horizontally instead of vertically."

$x =$ 4	2	9	-7	5	-19	2
		8	68	244	996	3908
	2	17	61	249	977	**3910**

"From left to right, each entry in the middle row is obtained by multiplying the preceding entry in the bottom row by the number being substituted in the polynomial — in this case, 4. Then the top row and middle row are added to get the next entry in the bottom row.

"The technique is called **Horner's Method** for evaluating polynomials. Another way of explaining is based on long division for polynomials, so it's also called **synthetic division**."

Use Horner's Method to evaluate the following polynomials:

1. $x^3 - 2x^2 + 5x - 9; \qquad x = 7$

2. $2x^4 - 9x^3 + 8x^2 + 17x - 4; \qquad x = -3$

3. $x^5 - 15x^3 + x; \qquad x = -1$

4. $3x^5 + x^4 + 7x^2 + 2; \qquad x = 9$

5. $x^5 - x^4 + x^3 - x^2 + x - 1; \qquad x = -1$