2. (a) \[ \begin{array}{c}
1 \quad 2 \\
2 \quad 4 \\
2 \quad 3 \\
2 \quad 1
\end{array} \]
(b) \[ \begin{array}{c}
1 \quad 2 \\
2 \quad 3 \\
1 \quad 2
\end{array} \]
(c) \( \chi(G) = 3 \) since there is a 3-coloring, and at least 3 are needed since graph contains \( K_3 \).

4. (a) \[ \begin{array}{c}
1 \quad 2 \\
3 \quad 4 \\
2 \quad 3 \\
1 \quad 4
\end{array} \]
(b) \( \chi(G) = 4 \). There is a 4-coloring, and there is no 3-coloring since graph contains \( K_4 \).

8. If any vertices are not isolated, then some vertices are adjacent to others. Any pair of adjacent vertices requires at least two colors.

10. If \( n \) is even, the outer circuit \( (C_n) \) requires 2 colors, so a third is needed for the "hub". So \( \chi(W_n) = 3 \) if \( n \) is even. Likewise if \( n \) is odd, \( \chi(C_n) = 3 \), so a fourth color is needed for the hub, so \( \chi(W_n) = 4 \) if \( n \) is odd.
18. (a) If every vertex is degree 3, then every vertex is odd. We know from before that a graph has an even number of odd vertices.

    n must be at least 4, since it is even (from (a)), and it is non-negative, so it is 0 or 2. If there are zero or two vertices, we can't have any vertices of degree 3, and all vertices are of degree 3. So n is an even number— that is not 0 or 2, so it is 4 or greater.

    (b) From Brooks' Theorem, \( \chi(G) \leq \max \text{ degree} + 1 \). Max degree is 3, so \( \chi(G) \leq 3 + 1 = 4 \).

    (c) \( K_4 \) fails

22. (a) Every vertex is adjacent to 8 others in the same box, 8 others in the same row, and 8 others in the same column. However, some are counted twice here—there are 4 vertices in the same box that are also in the same row/column. So only count those once, overall there are 20 neighbors.

    (b) \( \# \text{ edges} = \frac{\text{degree sum of } G}{2} \). There are 81 vertices, each has degree 20. So degree sum is 1620. \( \frac{1620}{2} = 810 = \# \text{ edges} \).