Test I

(1) Evaluate $\sin(\arcsin(0.26))$, $\cos(\arccos(0.26))$, $\sin(\arccos(0.26))$, $\arccos(\cos(\frac{7\pi}{5}))$, $\arcsin(\sin(\frac{7\pi}{5}))$, $\arcsin(\sin(10))$, $\sin(\arcsin(\sin(10)))$. See also (4.6;1-22) from textbook.

(2) Evaluate $\int \ln x \, dx$, $\int \arcsin(x) \, dx$.

(3) Derive the product formula for $\cos(a)\cos(b)$ and use that formula to evaluate $\int \cos(x)\cos(2x) \, dx$. Consider similar problems for $\cos(a)\sin(b)$, $\sin(a)\sin(b)$.

(4) Express $\int \sec(x) \, dx$ in terms of a function F which is the integral of a rational function.

(5) (For final exam) Derive the recursion formula for the integrals $\int \frac{1}{(x^2+1)^n} \, dx$.

Test II

By definition, a "trigonometric integral" is a function whose derivative is a rational function of $\sin(\theta)$ and $\cos(\theta)$.

(1) Expand $\frac{2x^2+5}{x^3+x^2+x-3}$ in partial fractions.

(2) Evaluate $\int \frac{2x+5}{x^2-4x+8} \, dx$.

(3) Evaluate $\int \frac{2x+5}{x^2-4x-8} \, dx$.

(4) Express $\int x\sqrt{10+3x+x^2} \, dx$ in terms of a trigonometric integral F.

(5) Express $\int x\sqrt{6-7x-2x^2} \, dx$ in terms of a trigonometric integral F.

Test III

(x) Expand $\ln|3x-6|$ in a power series about $a=1$.

(x) Expand the function $\frac{1}{(5+3x)^2}$ in a power series about $a=4$.

(x) Use the ratio test to determine the radius of convergence of the following power series:

$$\sum_{n=3}^{\infty} \frac{(-1)^n (x+4)^{5n}}{n^4 3^{n+1}}.$$

What is the largest open interval on which the series converges?

Final Exam

(1) (For final exam) Derive the recursion formula for the integrals $\int \frac{1}{(x^2+1)^n} \, dx$ in terms of a function F which is the integral of a rational function.

(x) Complete evaluation of algebraic integral.

(x) Evaluate $\int \sec(\theta) \, d\theta$ by use of subbstitution and partial fractions (no credit for formula alone or for use of method involving multiplication and division by $\sec(\theta)+\tan(\theta)$).

(x) Derive Taylor's formula, of order three, with remainder, for a function f about $a \in \mathbb{R}$.

(x) (i) State Taylor's formula, of order N, with remainder, for a function f about the center $a \in \mathbb{R}$. (ii) Write Tayor's polynomial $T_3(x)$ of order three about the center 0 for $\sin(x)$. (iii) Write the integral expression for the remainder in Taylor's formula of order three about the center 0 for $\sin(x)$. (iv) Use the remainder of part (iii) to estimate $|\sin(1) - T_3(1)|$.

(x) Derive the recursion formula for the integrals $\displaystyle\int \frac{dx}{(x^2+1)^n}$.

(x) (i) State the existence and uniqueness theorem for initial value problems for second order linear differential equations with constant coefficients. (ii) State a theorem which describes the form of a particular solution to a certain type of linear differential equation with constant cofficients. (iii) State a theorem which describes the general solutions of linear differential equations with constant cofficients. (iv) Define the expression "z is a root of $P(X)$ of multiplicity M". Give three particular pairs z, $P(X)$ exemplifying, respectively, roots of multiplicity 0, 1, 2. In each case choose $P(X)$ of degree 2.

(x) Give a general solution of each of the following homogeneous differential equations.
(i) $6y''(t) - 5y'(t) + 3y(t) = 0$.
(ii) $y''(t) + 2y'(t) - 5y(t) = 0$.
(iii) $y''(t) - \sqrt{28}y'(t) + 7y(t) = 0$. Evaluate $\displaystyle\int \sec(\theta)\, d\theta$

(x) For each of the following differential equations, give the *form* of a particular solution as prescribed by the theorem discussed in class. Do not solve for the coefficients.
(i) $y'(t) = t^2 + 6$.
(ii) $y'(t) + 5y(t) = (t^2+2)e^{-5t}$.
(iii) $y''(t) + 5y(t) = (t^2+3)e^{3t}\cos(t)$.

(x) Give a particular solution of the differential equation $y'(t) + y(t) = t\cos(t)$.

(x) Solve the initial value problem $y'(t) + 4y(t) = 12, \quad y(0) = 0$.

(x) Solve the initial value problem $y''(t) + 4y(t) = 8t$, $y(0) = 2$, $y'(0) = 0$.

(x) In each of the following problems, give the general solution of the homogeneous differential equation Use 0 as the particular solution in each case.
$5y''(t) - 6y'(t) + 6y(t) = 0$.
$3y''(t) + y'(t) - 5y(t) = 0$.
$5y''(t) - \sqrt{20}y'(t) + y(t) = 0$.
$5y''(t) = 0$.
$5y'(t) = 0$.

(x) For the following differential equation, give the *form* of a particular solution as prescribed by the theorem discussed in class. Do not solve for the coefficients.
$y'(t) + y(t) = (5 + 6t + t^2)e^{2t}\cos(t) + (t^3)e^{2t}\sin(t)$.
For 28 more problems of this sort see the pdf file "Particular Solutions of Differential Equations"

(x) Give a particular solution of the differential equation $y'(t) + y(t) = t\sin(t)$.

(x) Solve the initial value problem $y'(t) + 3y(t) = 15, \quad y(0) = 1$.

(x) Solve the initial value problem $y''(t) + 4y(t) = 4t$, $y(0) = 2$, $y'(0) = 0$.