Derivation of Taylor's formula of order 2 for the function f about $a \in \mathbb{R}$

$$
\begin{aligned}
f(x) = f(a) + \int_a^x f'(t)\,dt &= f(a) + \left.(t-x)f'(t)\right|_a^x - \int_a^x (t-x)f''(t)\,dt \\
&= f(a) + \left.(t-x)f'(t)\right|_a^x - \left[\left.\frac{(t-x)^2}{2} f''(t)\right|_a^x - \int_a^x \frac{(t-x)^2}{2} f'''(t)\,dt \right] \\
&= f(a) + (x-x)f'(x) - (a-x)f'(a) \\
&\quad - \left[\frac{(x-x)^2}{2} f''(x) - \frac{(a-x)^2}{2} f''(a) - \int_a^x \frac{(t-x)^2}{2} f'''(t)\,dt \right] \\
&= f(a) - (a-x)f'(a) + \frac{(a-x)^2}{2} f''(a) + \int_a^x \frac{(t-x)^2}{2} f'''(t)\,dt \\
&= f(a) + f'(a)(x-a) + f''(a)\frac{(x-a)^2}{2} + \int_a^x \frac{(x-t)^2}{2} f'''(t)\,dt
\end{aligned}
$$