Math 100-1 □ Survey of Mathematics □ Fall 2007

Exam # 1, Version A, September 10, 2007

The quadratic formula. The quadratic equation $Ax^2 + Bx + C = 0$ has the solutions

$$x = \frac{1}{2A}(-B \pm \sqrt{B^2 - 4AC}).$$

1. (2 points)

   \[
   \frac{1}{2} + \frac{1}{3} =
   \]

   (A) $\frac{2}{5}$ (B) $\frac{1}{6}$ (C) $\frac{5}{6}$ (D) $\frac{6}{5}$ (E) None of these

Answer: $5/6$, C

2. (2 points)

   \[
   \frac{1}{2} \cdot \frac{2}{3} =
   \]

   (A) $\frac{2}{5}$ (B) $\frac{1}{3}$ (C) $\frac{3}{5}$ (D) 3 (E) None of these

Answer: $1/3$, B

3. (2 points)

   \[
   \frac{1}{2} \div \frac{2}{3} =
   \]

   (A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{5}{2}$ (D) $\frac{1}{3}$ (E) None of these

Answer: $3/4$, A

4. (2 points) If

   $$6x + 32 = 60 - x$$

then

   (A) $x = 5$ (B) $x = 4$ (C) $x = 0$
   (D) $x = 2$ (E) None of these

Answer: 4, B
5. (2 points) If \[ 5x + 30 = 41 - \frac{1}{2}x \]
then
- (A) \( x = 5 \)
- (B) \( x = 4 \)
- (C) \( x = 3 \)
- (D) \( x = 2 \)
- (E) None of these

Answer: \( x = 2 \), D

6. (2 points)
\[ x^2 + 6x - 4 = \]
- (A) \((x + 6)^2 - 40\)
- (B) \((x + 3)^2 - 13\)
- (C) \((x + 3)^2 - 4\)
- (D) \((x - 3)^2 - 13\)
- (E) None of these

Answer: \((x + 3)^2 - 13\), B

7. (2 points) Add the labels of the true statements.

1. For all numbers \( a, b \), \((a + b)(a - b) = a^2 - b^2\).
2. For all numbers \( a, b \), \((a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3\).
3. For all numbers \( a, b \), \((a - b)^2 = a^2 - 2ab + b^2\).

- A. 5
- B. 7
- C. 10
- D. 14
- E. None of these.

Answer: All true, B

8. (2 points) Add the labels of the true statements. (Here \( a, b, c, d, e \) are any numbers and it is assumed that none of the denominators is zero.)

1. \( \frac{(a + 1)c}{5c} = \frac{a + 1}{5} \)
2. \( \frac{1 + bc}{1 + ec} = \frac{1 + b}{1 + e} \)
3. \( a + (2 \cdot c) = (a + 2) \cdot c \)
4. \( \frac{a}{b} \sqrt{\frac{c}{d}} = \frac{ad}{bc} \)

- A. 4
- B. 8
- C. 9
- D. 12
- E. None of these.

Answer: \( 1 + 8 = 9 \), C
9. (2 points) Add the labels of the true statements.

[1] The real numbers are exactly the positive or negative, finite or infinite decimal fractions.
[2] The natural numbers are the numbers needed for counting.
[4] The number 0 is not needed for counting. It was invented as a place holder in a place value system of enumeration, such as the decimal system.
[8] \( \mathbb{Q} \) is the standard symbol for the system of rational numbers.

A. 4 B. 8 C. 10 D. 12 E. None of these.

Answer: 1 + 2 + 4 + 8 = 15, E

10. (2 points) One solution of

\[
 n^2 + 5n - 6 = 0
\]

is

(A) \( n = 1 \)  \hspace{1cm} (B) \( n = -2 \)  \hspace{1cm} (C) \( n = 3 \)

(D) \( n = -1 \)  \hspace{1cm} (E) None of these is a solution

Answer: The solutions are 1 and \(-6\), A

11. (2 points) The solutions of

\[
 n^2 + 5n - 6 = 0
\]

are

(A) \(-6\)  \hspace{1cm} (B) \(1\) and \(-6\)  \hspace{1cm} (C) \(3\) and \(4\)

(D) \(-1\) and \(6\)  \hspace{1cm} (E) None of these

Answer: B

12. (2 points)

\[
 n^2 + 4n - 5 =
\]

(A) \((n - 1)(n + 5)\)  \hspace{1cm} (B) \((n + 1)(n - 5)\)  \hspace{1cm} (C) \((n - 3)^2\)

(D) \((n - 1)(n - 5)\)  \hspace{1cm} (E) None of these

Answer: A
13. (2 points)

\[ n^2 + n + 1 = \]

(A) \((n + \frac{1}{2})^2 + \frac{3}{4}\)
(B) \((n - \frac{1}{2})^2 + \frac{3}{4}\)
(C) \((n + \frac{1}{2})^2 + 1\)
(D) \((n + \frac{1}{2})^2 - \frac{1}{4}\)
(E) None of these

Answer: A

14. (2 points) The unique solution of the linear system

\[ 3x + y = 7, \quad x - y = 5, \quad \text{is} \]

(A) \(x = 1, y = -2\)
(B) \(x = -2, y = 1\)
(C) \(x = 3, y = -2\)
(D) \(x = 1, y = -1\)
(E) None of these

Answer: C

15. (2 points) The unique solution of the linear system

\[ x + y = 7, \quad x - y = 5 \quad \text{is} \]

(A) \(x = 1, y = 6\)
(B) \(x = 6, y = 1\)
(C) \(x = 3, y = 3\)
(D) \(x = 6, y = -1\)
(E) None of these

Answer: B

16. (2 points) A solution of

\[ (n + 1)^2 + 1 = 0 \quad \text{is} \]

(A) \(n = 0\)
(B) \(n = -1\)
(C) \(n = 1\)
(D) \(n = -2\)
(E) None of these

Answer: E

17. (2 points)

\[ (-3 + 7) - (2 + 3 - 4) + (2(3 - 4) - 5)) = \]

(A) 4
(B) -6
(C) 0
(D) -4
(E) None of these

Answer: 4 - 1 + (-2 - 5) = 3 - 7 = -4, D
18. (2 points) Add the labels of the true statements. (Here $a, b, c, d, e$ are any numbers and it is assumed that none of the denominators is zero.)

\[ [1] \frac{1}{c} \frac{a}{b} = \frac{a}{bc} \quad [2] \frac{a}{b} \frac{c}{b} = \frac{a}{c} \quad [4] \frac{a}{c} \frac{c}{a} = \frac{a^2}{bc} \]

A. 4   B. 5   C. 6   D. 7   E. None of these.

Answer: All true, D

19. (2 points) A pizza is cut into pieces. Albert gets $\frac{1}{6}$, Fred gets $\frac{2}{5}$, and George gets $\frac{1}{3}$. How much is left for Jon?

\[ (A) \frac{1}{10} \quad (B) \frac{1}{5} \quad (C) \frac{1}{6} \quad (D) \frac{1}{12} \quad (E) \text{ None of these} \]

Answer: $\frac{1}{6} + \frac{2}{5} + \frac{1}{3} = \frac{1}{30}(5 + 12 + 10) = 27/30 = 9/10$, A

20. (2 points) If

\[ x - \frac{1}{3}x = x + 1, \]

then

\[ (A) \quad x = \frac{2}{3} \quad (B) \quad x = -\frac{1}{3} \quad (C) \quad x = 3 \]
\[ (D) \quad x = -3 \quad (E) \text{ None of these} \]

Answer: D

21. (2 points) If

\[ x + 2x - 3x - 4x + 5x = 2x - 4x + 3x \]

then, necessarily,

\[ (A) \quad x = 0 \quad (B) \quad x = 1 \quad (C) \quad x = -1 \]
\[ (D) \quad x = 2 \quad (E) \text{ None of these} \]

Answer: Simplifies to $x = x$ which is true for all $x$, E
22. (2 points) The volume $V$ of a sphere with radius $r$ is given approximately by the formula $V = \frac{4}{3}\pi r^3$. If $V = \frac{11}{21}$ cubic meters, then $r$ is approximately

- (A) $\frac{1}{2}$ meter
- (B) $\frac{1}{3}$ meter
- (C) $\frac{1}{4}$ meter
- (D) $\frac{1}{8}$ meter
- (E) None of these

Answer: A

23. (2 points) The formula that relates the measurement in centigrades $C$ and in Fahrenheits $F$ of a temperature is

$$F = 32 + \frac{9}{5}C.$$  

If a certain temperature measures 71 degrees in Fahrenheits, what is its measure in centigrades rounded to full degrees?

- (A) 21
- (B) 22
- (C) 23
- (D) 24
- (E) None of these

Answer: $71 = 32 + \frac{9}{5}C$, $C = 21 + \frac{2}{3}$, B

24. (2 points) The quadratic equation

$$6x^2 - 5x + 1 = 0$$

has the solutions

- (A) $-\frac{1}{2}$ and $-\frac{1}{3}$
- (B) $-\frac{1}{2}$ and $\frac{1}{3}$
- (C) $\frac{1}{2}$ and $-\frac{1}{3}$
- (D) $\frac{1}{2}$ and $\frac{1}{3}$
- (E) None of these

Answer: D

25. (2 points) The quadratic equation

$$(x - 2)\left(x + \frac{1}{2}\right) = 0$$

has the solutions

- (A) $\frac{1}{2}$ and 2
- (B) $-\frac{1}{2}$ and $-2$
- (C) $-\frac{1}{2}$ and 1
- (D) $-\frac{1}{2}$ and 2
- (E) None of these

Answer: D