

THE LOOSE LESSONS OF CALCULUS (FIRST DRAFT)

1. NEAR AND FAR

- (1) The limit of a function at a point is the behavior of the values of a function near the point.
- (2) Infinite limits are long-term behavior or far away behavior of a function.
- (3) If the value of a function at a point is the same as its behavior near the point, then the function is continuous at that point.

2. QUANTIFYING CHANGE

- (1) The derivative of a function at a point is all three of the following:
 - (a) The slope of the tangent line to the function at the point.
 - (b) The rate of change of the function at a single point in time.
 - (c) The limit of the average rates of change around the point i.e. the behavior of the average rates of change near the point.
- (2) If a function has a tangent line at a point, then the value of the function at the point is the same as its behavior near the point.
- (3) The tangent line approximates the function near the point i.e. the tangent line at a point is close to the function near the point.

3. AREA

- (1) The integral of a function on closed interval is the area beneath the function on that closed interval.
- (2) The area under a curve can be approximated by filling the space with many rectangles and adding up the area of the rectangles.
- (3) The area under the velocity (rate of change) curve on a closed interval is the displacement of the function on that interval.

4. APPLICATIONS

- (1) If the value of a function f is dependent on the value of a function g , then the rate of change of f at a point in time is

$$(\text{rate of change of } f \text{ at } g) \times (\text{rate of change of } g \text{ at } t)$$

- (2) Suppose f is continuous at a point a . If the function is decreasing on the entire left of a and increasing on the entire right of a , then f has its smallest value at a . If the function is increasing on the entire left of a and decreasing on the entire right of a , then f has its largest value at a .