DOLD SEQUENCES OVERVIEW

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Let $s : \mathbb{N} \to \mathbb{Z}$ be an integer sequence. To this sequence we associate another sequence, $M_s : \mathbb{N} \to \mathbb{Z}$, called the Möbius inversion sequence. It is defined by the following equation:

$$M_s(n) = \sum_{d|n} \mu(d) s\left(\frac{n}{d}\right)$$

where μ is the Möbius function. If $k|M_s(k)$ for all $k \in \mathbb{N}$, we call $s : \mathbb{N} \to \mathbb{Z}$ a Dold sequence.

Sequences satisfying this property have nice applications in periodic point theory. Let X be a Euclidean Neighborhood Retract (e.g. a simplicial complex or a manifold) and $f : X \to X$ a self-map. We will denote the Lefshetz number (over $\mathbb{Q}$) of f by $\Lambda(f)$. Then $\Lambda(f)$ is an integer and the sequence

$$(\Lambda(f),\, \Lambda(f^2), \Lambda(f^3), \ldots)$$

is an integer sequence. Albrecht Dold proved that if the fixed point set of f^k is compact (e.g. if X itself is compact) for every k, then the sequence of Lefshetz iterates is a Dold sequence. Now, recall that a periodic point of f is a point $x \in X$ such that $f^k(x) = x$ for some $k > 1$ but $f^j(x) \neq x$ for any $0 < j < k$. Dold's theorem intuitively tells us that every periodic point of period k should have k points in its orbit under the group generated by f. This interpretation unfortuneately fails most of the time because one cannot always interpret the Lefshetz number of a map as the number of fixed points of that map. Nonetheless, we still have the divisibility property of the sequence which helps to close the gap between inuition and actuality.

To this end, I am interested in determining the number theory of Dold sequences when the space X is a closed manifold. Since the Lefshetz number is an isotopy invariant, the hope is that this will reveal new information about mapping class groups. To aid in this goal, I have studied Dold sequences from an abstract perspective. For example, what can one say when the Dold sequence is also periodic? Surprisingly, this number theoretic characterization encodes interesting information about a periodic map from a space to itself.

For maps that are not periodic, some authors have worked out an asymptotic theory. In fact, it has been shown somewhat recently that an unbounded Dold sequence is asymptotic to the exponential function. An interesting corollary of my work on periodic Dold sequences shows that a bounded Dold sequence is necessarily periodic. It seems as though all of this might point in the direction of a topological version of the Shub-Sullivan theorem. However, to do this, it is necessary to understand what hypotheses on X force the Möbius inverse sequence to detect the periodic points of self-maps on X.

If you would like details about this interesting topic, please e-mail me.