Math 321 Take-home Midterm Exam #2

Directions: You may use your notes, homework, course textbook, and a calculator to help you with these problems. You may not reference any other books or the Internet for help. You may not discuss these problems with each other or with anyone else before turning them in. You may email me with questions or requests for clarification, but I reserve the right not to respond if I think the question is clear.

The test is due when you walk into class on Thursday, April 9. No late tests will be accepted for any reason other than a documented medical excuse. If you are going to miss class on Thursday, it is your responsibility to have the test delivered to me somehow before 9am on Thursday.

Section 1. Choose one of the following two problems, 10 points.

1. Let $A$ and $B$ be sets. Prove:
   $$[(A \subseteq B) \lor (B \subseteq A)] \iff [\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)].$$
   (Here $\mathcal{P}(X)$ represents the power set of the set $X$.)

2. Let $A$ and $B$ be sets in some universe $U$. Recall we defined the symmetric difference of $A$ and $B$ as
   $$A \triangle B = (A \setminus B) \cup (B \setminus A).$$
   Prove: $A \triangle B = A^c \cup B^c$. 

Section 2. Choose one of the following two problems, 20 points.

3. Let $A$ and $B$ be sets in some universe $U$. Prove that the following are equivalent:
   (a) $A \subseteq B$.
   (b) $A^c \cup B = U$.
   (c) $A \cap B^c = \emptyset$.

4. Recall the definitions:
   **Well-ordering Principle on** $\mathbb{N}$: Every nonempty subset of $\mathbb{N}$ has a smallest element.
   **Principle of Mathematical Induction**: Consider $S \subseteq \mathbb{N}$. If $1 \in S$ and $n + 1 \in S$ whenever $n \in S$, then $S = \mathbb{N}$.
   Prove that these two principles are equivalent. That is, prove that WOP holds if and only if induction holds.

Optional Extra Credit You may choose to do one additional problem from either section above, for a maximum of 10 extra credit points.