Numerical Problems
1. Find all $x$ in $\mathbb{Z}_{12}$ which solve the equation $8x = 4$ in $\mathbb{Z}_{12}$. Find all $x$ in $\mathbb{Z}_{55}$ which solve the equation $15x = 35$ in $\mathbb{Z}_{55}$. Find all $x$ in $\mathbb{Z}_{531}$ which solve the equation $186x = 221$ in $\mathbb{Z}_{531}$.

2. ⊛ $U_8 = \{1, 3, 5, 7\}$. What set do we get, mod 8, when we multiply each element of $U_8$ by 3? By 5? By 7? $U_9 = \{1, 2, 4, 5, 7, 8\}$. What set do we get, mod 9, when we multiply each element of $U_9$ by 2? By 4? By 5? By 7? By 8? Do the same for $U_{10}$ and $U_{11}$. Any conjectures?

3. Does $3 + i$ divide $-1 + 7i$? Does $3 + 5i$ divide $21 + i$? Justify. Describe all of the “integral” multiples $(1+2i)a$ of $1+2i$. Here $a$ runs through all the Gaussian integers. Plot these “integral” multiples in the plane.

4. Calculate $\varphi(21)$, $\varphi(33)$, $\varphi(35)$, $\varphi(39)$, $\varphi(51)$, $\varphi(55)$, $\varphi(65)$, and $\varphi(77)$. Any conjectures? Hint: for each $m$, prime factor both $m$ and $\varphi(m)$.

Exploration
5. ⊛ Under what conditions on $a, b, c$ does the equation $ax + by = c$ in $\mathbb{Z}$ have a solution? Do enough examples until you can formulate a precise statement. You do not need to prove your statement (yet).

Prove or Disprove and Salvage if Possible (PODASIP)
6. ⊛ If $a|c$ and $b|c$ then $ab|c$.

7. ⊛ The only units in $\mathbb{Z}$ are $\pm 1$.

8. The product of three consecutive integers is divisible by 6.

9. Let $a$ and $n$ be positive integers $> 1$. If $a^n - 1$ is prime then $a = 2$ and $n$ is prime.

For Fun
10. Consider a rectangular Cartesian coordinate system in a plane. Points with integral coordinates we call lattice points. Show that no triangle whose vertices are all lattice points can be an equilateral triangle.