Math 613: Group Theory, Fall 2009

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Office Hours:	TBA, and by appointment
Class Meeting:	MWF, 11:30 – 12:20am in Keller 414

Course Description and Goals: My hope is to cover an array of topics that are useful in modern research in various fields. Rather than studying groups from a purely abstract perspective, we will look at ideas emerge in the study of specific groups that mathematicians find themselves working with all the time. Here are some topics I plan to cover in lecture, though we may not get to all of them. Others can be among the topics of the final projects and presentations.

Solvable Groups: Group theory really grew out of the proof of the inability to find a "quintic formula": something like the quadratic formula which would allow one to find the roots of any polynomial of the form $f(x) = a_0x^5 + a_1x^4 + a_2x^3 + a_3x^2 + a_4x + a_5$. We will start with a *quick* review of some background material, then study solvable groups and the classic proof of the insolubility of the quintic.

(Linear) Representations of Finite Groups: The idea behind group representations is to describe arbitrary groups in terms of linear transformations of vector spaces, which we understand quite well (think matrix multiplication). These techniques have had a huge impact on modern research, for example in the classification of finite simple groups.

Topological Groups and Profinite Groups: These are the natural objects of study after finite groups. After describing the construction of profinite groups via direct limits (and other methods), we can explore the representation theory of compact topological groups, a little bit of the theory of Lie groups, and possibly some of the Galois theory of infinite field extensions.

Group Cohomology: Group cohomology grew out of both topology (where certain homological properties of a space seemed to depend only on its fundamental group) and number theory (as a device for describing the main theorems of class field theory). It is in some ways a natural extension of the idea of group representations.

Elliptic Curves: These provide an interesting example of groups that appear in algebraic geometry. We could study some of the elementary properties of these curves. If time permits, we can tie several ideas together with the study of ℓ-adic representations coming from elliptic curves.

Text: The book *Group Theory* by W.R. Scott has been ordered and will be in available in the bookstore. However, it covers very few of the topics above. No book will be required, though you will likely find it useful to have some kind of reasonable reference on hand. Scott's book is a good (an inexpensive) choice.

I will occasionally hand out notes or point you to resources on the web. But the upshot is that you'll have to come to class, study your notes, and work through problems in order to be successful in this course.

Grading: I will regularly assign problems in class. I hope that we can schedule a problem session in addition to our regular class meeting, where students will take turns presenting the problems to each other. I will collect write-ups of a few of the problems to grade over the course of the semester. There will be a final project, where students will research a topic in group theory that we haven't covered, write a short expository paper on the topic, and present about the topic to the class. Your grade will be based on participation in the problem sessions, participation in class, the occasional problem write-ups, and (mostly) the final project.

Note that this is an advanced-level graduate class. My expectation is that you will only take the course if you are interested in learning the material and are willing to put in some time and effort to do so. And anyone who puts reasonable time and effort into an advanced graduate course should expect to earn an A or a B.