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p-adic Rep's of $Gal(\mathbb{Q}_p)$ to Sheaves on Flag Varieties

Joint work w/ P. Schneider & G. Zabradi

Start w/ p-adic rep'n of $Gal(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$ $\xrightarrow[\text{construct}]{\text{want to}}$ G-invariant sheaves on G/P

$G = GL(n, \mathbb{Q}_p)$ $n \geq 1$

$P = \begin{pmatrix} \times & & * \\ & \ddots & \\ 0 & & \times \end{pmatrix}$

G/P given p-adic topology

Motivation: p-adic Langlands correspondence

$n=1$: Local Class Field Theory

$L/\mathbb{Q}_p$ finite extension $\quad Gal(\overline{\mathbb{Q}}_p/\mathbb{Q}_p) \longrightarrow O_L^\times$

$$\mathbb{Q}_p^\times \longrightarrow O_L^\times$$

Want to generalize this to $n > 1$.

$\{$Continuous rep'ns of $Gal(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$ of L-vector spaces of dim 2$\}$ $\quad g_2 \quad g_2^{irr}$

$\updownarrow$ Colmez Corresp.

$\{$cont. rep'n of $GL(2,\mathbb{Q}_p)$ on L Banach spaces$\}$ admissible, central character $\quad B_2 \quad B_2^{irr}$

$p \neq 2,3$, image = everything "not ordinary"

$n \geq 3$? Nothing is known.

Goal: Generalize construction of Colmez

note: finitely generated O_L-modules denoted g_{O_L}

$g_{O_L} \simeq (\varphi, T)$-modules étale and fin. gen. over

$L^* \circlearrowright O_\varepsilon = \left(O_L[[N_{2,0}]] \text{ elt's not div by } p\right)^\wedge$ ← p-adic completion

$L^* \circlearrowright N_{2,0} = \begin{pmatrix} 1 & \mathbb{Z}_p \\ 0 & 1 \end{pmatrix}$

$L^* = \begin{pmatrix} \mathbb{Z}_p \setminus \{0\} & 0 \\ 0 & 1 \end{pmatrix}$

fin. gen. O_ε-module D

semi-linear action of L^*

φ action of $\begin{pmatrix} p & \\ & 1 \end{pmatrix}$

φ injective $\quad D = \bigoplus_{i=0}^{p-1} \begin{pmatrix} 1 & i \\ 0 & 1 \end{pmatrix} \varphi(D) \quad \varphi \leftrightarrow \begin{pmatrix} p & \\ & 1 \end{pmatrix} \quad T \leftrightarrow \begin{pmatrix} \mathbb{Z}_p^\times & 0 \\ 0 & 1 \end{pmatrix}$

$g_{O_L} \simeq \mathcal{M}_{O_\varepsilon}^{et}(L_2^*)$ $\qquad$ Center $L_2^+ = L_2^* \mathbb{Q}_p^\times$

Theorem: $\exists$ functor $\mathcal{M}^{\text{ét}}_{O_\mathcal{E}}(L_2^+) \longrightarrow \left\{\begin{array}{l} GL(2,\mathbb{Q}_p)\text{ - invariant} \\ \text{sheaves on } \mathbb{P}^1(\mathbb{Q}_p) \end{array}\right\}$

$G \qquad\qquad GL(n,\mathbb{Q}_p)$

$P = LN \qquad L = \begin{pmatrix} \times & & 0 \\ & \ddots & \\ 0 & & \times \end{pmatrix} \qquad N = \begin{pmatrix} 1 & & * \\ & \ddots & \\ 0 & & 1 \end{pmatrix}$

$N = \prod_{\alpha>0} N_\alpha \qquad N_\alpha \simeq \mathbb{Q}_p$

$\mu = \prod \mu_\alpha \qquad \mu_\alpha \in \mathbb{Q}_p$

$L^+ = \{t \in L \quad v_p\, \alpha(t) \geq 0 \text{ all simple roots } \alpha\}$

L^{++} strict inequality

$L \xrightarrow{\alpha} \mathbb{Q}_p^\times$

$L^+ \xrightarrow{\alpha} L_2^+ \qquad \underset{L^+}{\circlearrowleft} O_\mathcal{E}$

Th: $\mathcal{M}^{\text{ét}}_{O_\mathcal{E}}(L^+) \longrightarrow \left\{\begin{array}{l} \text{a family of } G\text{-equiv sheaves} \\ \text{of } O_L\text{-modules on } G/P \\ \text{indexed by } s \in L^{++} \end{array}\right.$

add an action of $L^+ \cap L^{\alpha=1}$

Th: $\exists$ functor $\mathcal{M}^{\text{ét}}_{O_\mathcal{E}}(L_2^+) \longrightarrow \left\{\begin{array}{l} GL(2,\mathbb{Q}_p)\text{-equivariant} \\ \text{sheaves on } \mathbb{P}^1(\mathbb{Q}_p) \\ \text{of } O_L\text{-modules} \end{array}\right.$

$\uparrow$

g_2, O_L