The probability that a curve over a finite field is smooth
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Given a fixed surface over a finite field, we ask what proportion of curves in that surface are smooth. Poonen’s work on Bertini theorems over finite fields answers this question for certain families of curves in the surface. In this case the probability of smoothness is predicted by a simple heuristic assuming smoothness is independent at different points in the surface. In joint work with Erman, we consider this question for other families of curves in $P^1 \times P^1$ and Hirzebruch surfaces. Here the simple heuristic of independence fails, but the answer can still be determined and follows from a richer heuristic that predicts at which points smoothness is independent and at which points it is dependent.