Melanie

Prob that a curve over $\mathbb{F}_q$ smooth

$$\lim_{d \to \infty} \frac{\# C \text{ smooth deg } d}{\# C \text{ deg } d}$$

C def by $\{f = 0\}$ $\quad$ $f = \frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = \frac{\partial f}{\partial z} = 0$ $\quad$ no sol's over any $\mathbb{F}_{q^r}$

Heuristic: Consider $s \in \mathbb{P}^2$ def/ $\mathbb{F}_q$ e.g. $s = [0:0:1]$

$f = a + bx + cy + \text{h.o.t.}$

C sing $\Leftrightarrow a = b = c = 0$ $\quad$ guess: prob $= \frac{1}{q^3}$

similarly, guess $\frac{1}{q^{3d}}$ fo pt s of deg d

Guess: $\text{Prob(smooth)} \overset{?1}{=} \prod_{s \in \mathbb{P}^2} \text{Prob}(C \text{ smooth at } s)$

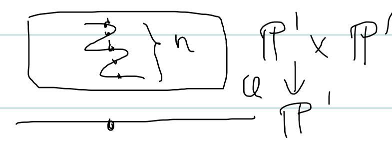
$$\overset{?2}{=} \prod_{s \in \mathbb{P}^2} \left(1 - \frac{1}{q^{3\deg(s)}}\right)$$

?2: Easy computation,
factors don't depend on $d >> 0$

?1: Hard Thm of Poonen
(Bertini Thms over $\mathbb{F}_q$)

Curves in $\mathbb{P}^1 \times \mathbb{P}^1$ have bidegree (d_1, d_2)

1) deg $\to \infty$ on line thru origin w/ $0 < m < \infty$
done by applying Poonen's Thm

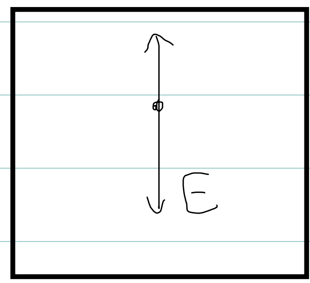


2) deg $\to \infty$ as (n, d) $\quad$ n fixed, $d \to \infty$

At pts in a fiber of u
Smoothness can't be ind.
e.g. one can control # sing pts in terms of n

<u>Thm</u>: $\text{Prob}(C \text{ smooth})_{(n, d \to \infty)} = \prod_{\text{fibers of } u} \text{Prob}\underbrace{\begin{pmatrix} C \text{ smooth at pts} \\ \text{in that fiber} \end{pmatrix}}$

don't depend on $d >> 0$
easy to compute



$Bl_{\{pt\}}\mathbb{P}^2$

Curve bidegree (d_1, d_2)

$\cap$

$E = d_2$ pts

d_2 fixed

$d_1 \to \infty$