Math 100 – worksheet 11 –

Doubling time and half-life

1. (exercise 55 on p.491 from the book)
   A nation of 100 million people is growing at a rate of 4% per year. What will its population be in 30 years?

   We solve this question in several different ways and compare all answers.

   - We want to use the formula

     $$\text{new value} = \text{initial value} \times 2^{t/T_{\text{double}}}$$

     with the initial value of 100 million and period of time $t = 30$ years. We thus need T_{double}, and there are two ways to calculate it.

     i. Use the approximate doubling time formula to find T_{double}.

     ii. Find the population in 30 years using this value of T_{double}. What is a reasonable way to round this number?

     iii. Use the exaxt doubling time formula to find T_{double}.

     iv. Find the population in 30 years using this value of T_{double}. What is a reasonable way to round this number?

   - We now avoid using T_{double} all together. The yearly growth rate of 4% means that the population is multiplied by a factor of 1.04 every year. By which factor the population is multiplied over the period of 30 years? Find the population in 30 years using multiplication by this factor.

- Compare all three answers obtained.
  i. What do we conclude if we decide that 100 million in this question has one significant digit?

  ii. What do we conclude if we decide that 100 million in this question has three significant digit?

  iii. Pay attention at amazing closedness of two answers (much beyond the precision we need). The exact doubling time formula looks really exact. At the same time, the approximation with rule of 70 looks not at all bad.

2. (exercise 55 on p.491 from the book)
   The US government spent nearly \$10 billion planning a developing a nuclear waste facility in Yucca Mountain (Nevada), though the project was canceled in 2011 (update: deliberations about its renewal are still on the way).
   The intent has been for the facility to store up to $77,000$ metric tons of nuclear wast safely for at least 1 million years.
   Suppose the project goes ahead and stores the maximum amount of wast in the form of plutonium-239 with a half-life of $24,000$ years. How much plutonium would have remained after 1 million years?

   Make use of the formula
   $$\text{new value} = \text{initial value} \times \left(\frac{1}{2}\right)^{t/T_{\text{half}}}$$
   and put your answer in grams so that you can figure out the amount.