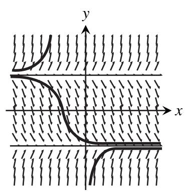
ANSWERS

CHAPTER 14

Section 14.1, pp. 14-8 to 14-9

1. (d) **3.** (a)

5.



7. $y' = x - y;\ y(1) = -1$ **9.** $y' = -(1 + y)\sin x;\ y(0) = 2$

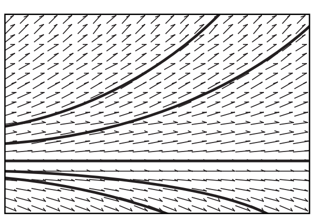
11. $y_0 = 2;\ y_1 = (1/2)x^2 + 3/2;\ y_2 = (1/2)x^2 + 3/2;$
$y_3 = (1/2)x^2 + 3/2$

13. $y_0 = 1;\ y_1 = (1/2)x^2 + 1/2;$
$y_2 = 5/8 + (1/4)x^2 + (1/8)x^4;$
$y_3 = 29/48 + (5/16)x^2 + (1/16)x^4 + (1/48)x^6$

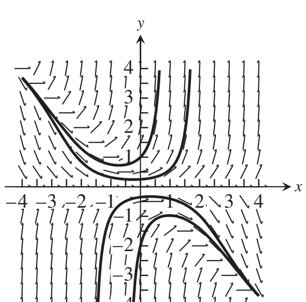
15. $y_0 = 1;\ y_1 - 1 + x + (1/2)x^2;$
$y_2 = 1 + x + x^2 + (1/6)x^3;$
$y_3 = 1 + x + x^2 + (1/3)x^3 + (1/24)x^4$

17. $y = (x_0 + y_0 + 1)e^{x - x_0} - (x + 1)$

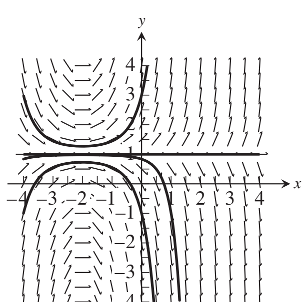
19.



21.



23.



Section 14.2, pp. 14-17 to 14-18

1. $y = \dfrac{e^x + C}{x},\quad x > 0$ **3.** $y = \dfrac{C - \cos x}{x^3},\quad x > 0$

5. $y = \dfrac{1}{2} - \dfrac{1}{x} + \dfrac{C}{x^2},\quad x > 0$ **7.** $y = \dfrac{1}{2}xe^{x/2} + Ce^{x/2}$

9. $y = x(\ln x)^2 + Cx$

11. $s = \dfrac{t^3}{3(t-1)^4} - \dfrac{t}{(t-1)^4} + \dfrac{C}{(t-1)^4}$

13. $r = (\csc\theta)(\ln|\sec\theta| + C),\quad 0 < \theta < \pi/2$

15. $y = \dfrac{3}{2} - \dfrac{1}{2}e^{-2t}$ **17.** $y = -\dfrac{1}{\theta}\cos\theta + \dfrac{\pi}{2\theta}$

19. $y = 6e^{x^2} - \dfrac{e^{x^2}}{x + 1}$ **21.** $y = y_0 e^{kt}$

23. **(b)** is correct, but **(a)** is not.

25. **(a)** 10 lb/min **(b)** $(100 + t)$ gal **(c)** $4\left(\dfrac{y}{100 + t}\right)$ lb/min

(d) $\dfrac{dy}{dt} = 10 - \dfrac{4y}{100 + t},\quad y(0) = 50,$

$$y = 2(100 + t) - \frac{150}{\left(1 + \frac{t}{100}\right)^4}$$

(e) Concentration $= \dfrac{y(25)}{\text{amt. brine in tank}} = \dfrac{188.6}{125} \approx 1.5$ lb/gal

27. $y(27.8) \approx 14.8$ lb, $t \approx 27.8$ min **29.** $t = \dfrac{L}{R}\ln 2$ sec

31. **(a)** $i = \dfrac{V}{R} - \dfrac{V}{R}e^{-3} = \dfrac{V}{R}(1 - e^{-3}) \approx 0.95\dfrac{V}{R}$ amp **(b)** 86%

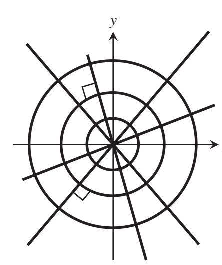
33. $y = \dfrac{1}{1 + Ce^{-x}}$ **35.** $y^3 = 1 + Cx^{-3}$

Section 14.3, pp. 14-22 to 14-23

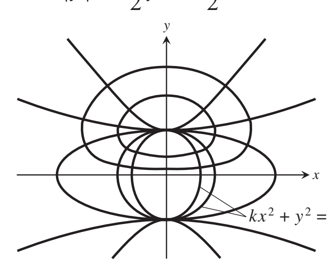
1. **(a)** 168.5 m **(b)** 41.13 sec

3. $s(t) = 4.91\left(1 - e^{-(22.36/39.92)t}\right)$

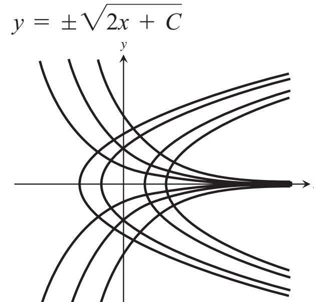
5. $x^2 + y^2 = C$



7. $\ln|y| - \dfrac{1}{2}y^2 = \dfrac{1}{2}x^2 + C$



9. $y = \pm\sqrt{2x + C}$



Section 14.4, pp. 14-27 to 14-28

1. $y\,(\text{exact}) = \dfrac{x}{2} - \dfrac{4}{x},\ y_1 = -0.25,\ y_2 = 0.3,\ y_3 = 0.75$

3. $y\,(\text{exact}) = 3e^{x(x+2)},\ y_1 = 4.2,\ y_2 = 6.216,\ y_3 = 9.697$

5. $y\,(\text{exact}) = e^{x^2} + 1,\ y_1 = 2.0,\ y_2 = 2.0202,\ y_3 = 2.0618$

7. $y \approx 2.48832$, exact value is e

9. $y \approx -0.2272$, exact value is $1/\left(1 - 2\sqrt{5}\right) \approx -0.2880$

11.

x	z	y-approx.	y-exact	Error
0	1	3	3	0
0.2	4.2	4.608	4.658122	0.050122
0.4	6.81984	7.623475	7.835089	0.211614
0.6	11.89262	13.56369	14.27646	0.712777

13. Euler's method gives $y \approx 3.45835$; the exact solution is $y = 1 + e \approx 3.71828$

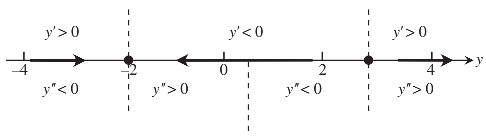
15. $y \approx 1.5000$; exact value is 1.5275.

17. **(a)** $y = \dfrac{1}{x^2 - 2x + 2},\quad y(3) = -0.2$
- **(b)** -0.1851, error ≈ 0.0149
- **(c)** -0.1929, error ≈ 0.0071
- **(d)** -0.1965, error ≈ 0.0035

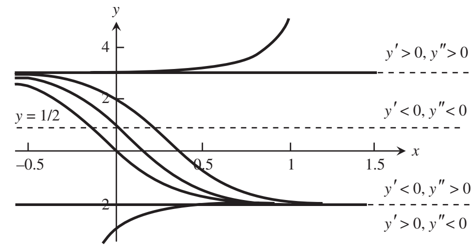
19. The exact solution is $y = \dfrac{1}{x^2 - 2x + 2}$, so $y(3) = -0.2$. To find the approximation, let $z_n = y_{n-1} + 2y_{n-1}(x_{n-1} - 1)\,dx$ and $y_n = y_{n-1} + \left(y_{n-1}^2(x_{n-1} - 1) + z_n^2(x_n^2 - 1)\right)dx$ with initial values $x_0 = 2$ and $y_0 = -\dfrac{1}{2}$. Use a spreadsheet, calculator, or CAS as indicated in parts (a) through (d).
- **(a)** -0.2024, error ≈ 0.0024
- **(b)** -0.2005, error ≈ 0.0005
- **(c)** -0.2001, error ≈ 0.0001
- **(d)** Each time the step size is cut in half, the error is reduced to approximately one-fourth of what it was for the larger step size.

Section 14.5, pp. 14-35 to 14-36

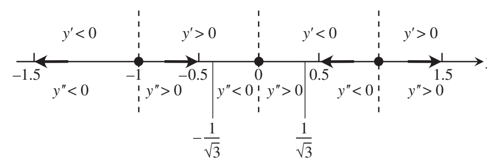
1. $y' = (y + 2)(y - 3)$
- **(a)** $y = -2$ is a stable equilibrium value and $y = 3$ is an unstable equilibrium.
- **(b)** $y'' = 2(y + 2)\left(y - \dfrac{1}{2}\right)(y - 3)$



(c)



3. $y' = y^3 - y = (y + 1)y(y - 1)$
- **(a)** $y = -1$ and $y = 1$ are unstable equilibria and $y = 0$ is a stable equilibrium.
- **(b)** $y'' = (3y^2 - 1)y' = 3(y + 1)\left(y + 1/\sqrt{3}\right)y\left(y - 1/\sqrt{3}\right)(y - 1)$



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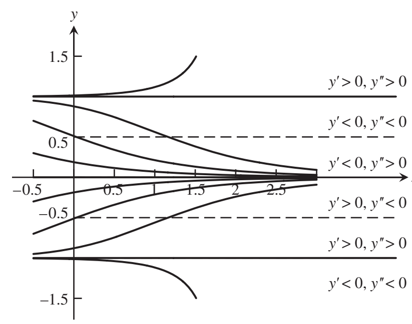
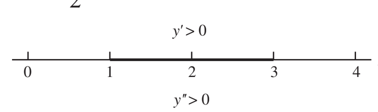


5. $y' = \sqrt{y},\ y > 0$
- **(a)** There are no equilibrium values.
- **(b)** $y'' = \dfrac{1}{2}$



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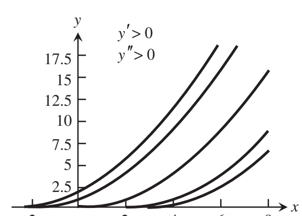
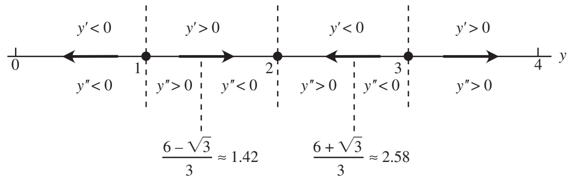


7. $y' = (y - 1)(y - 2)(y - 3)$
- **(a)** $y = 1$ and $y = 3$ are unstable equilibria and $y = 2$ is a stable equilibrium.
- **(b)** $y'' = (3y^2 - 12y + 11)(y - 1)(y - 2)(y - 3) =$

$$(y - 1)\left(y - \frac{6 - \sqrt{3}}{3}\right)(y - 2)\left(y - \frac{6 + \sqrt{3}}{3}\right)(y - 3)$$



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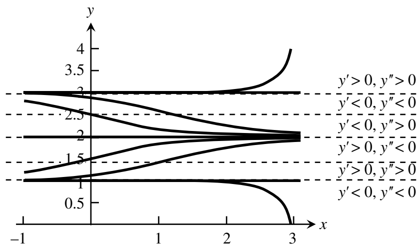
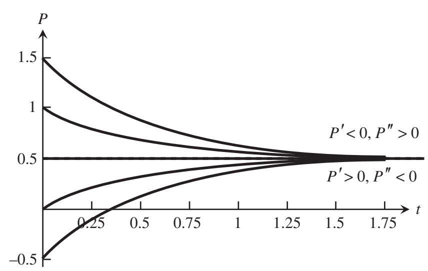
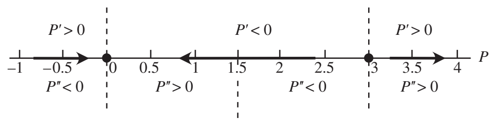
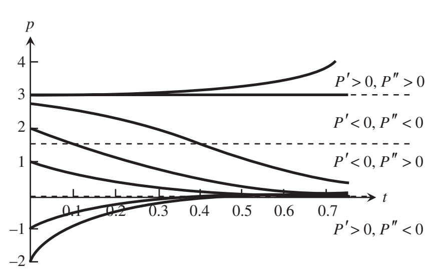


9. $\frac{dP}{dt} = 1 - 2P$ has a stable equilibrium at $P = \frac{1}{2}$; $\frac{d^2P}{dt^2} = -2\frac{dP}{dt} = -2(1 - 2P)$.

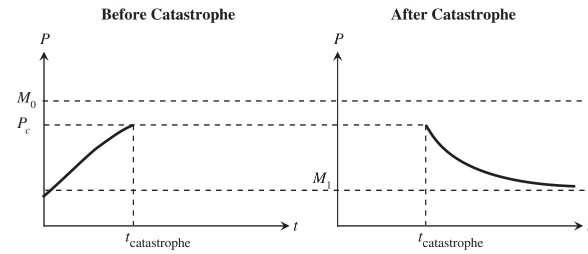


11. $\frac{dP}{dt} = 2P(P - 3)$ has a stable equilibrium at $P = 0$ and an unstable equilibrium at $P = 3$; $\frac{d^2P}{dt^2} = 2(2P - 3)\frac{dP}{dt} = 4P(2P - 3)(P - 3)$





13. Before the catastrophe, the population exhibits logistic growth and $P(t)$ increases toward M_0, the stable equilibrium. After the catastrophe, the population declines logistically and $P(t)$ decreases toward M_1, the new stable equilibrium.

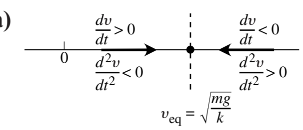


15. $\frac{dv}{dt} = g - \frac{k}{m}v^2, \quad g, k, m > 0$ and $v(t) \geq 0$

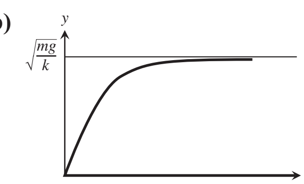
Equilibrium: $\frac{dv}{dt} = g - \frac{k}{m}v^2 = 0 \Rightarrow v = \sqrt{\frac{mg}{k}}$

Concavity: $\frac{d^2v}{dt^2} = -2\left(\frac{k}{m}v\right)\frac{dv}{dt} = -2\left(\frac{k}{m}v\right)\left(g - \frac{k}{m}v^2\right)$

(a)



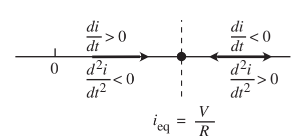
(b)



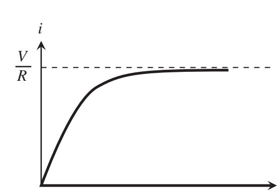
(c) $v_{terminal} = \sqrt{\frac{160}{0.005}} = 178.9$ ft/sec $= 122$ mph

17. $F = F_p - F_r$; $ma = 50 - 5|v|$; $\frac{dv}{dt} = \frac{1}{m}(50 - 5|v|)$. The maximum velocity occurs when $\frac{dv}{dt} = 0$ or $v = 10$ ft/sec.

19. Phase line:



If the switch is closed at $t = 0$, then $i(0) = 0$, and the graph of the solution looks like this:



As $t \to \infty$, $i(t) \to i_{steady\ state} = \frac{V}{R}$.

Section 14.6, pp. 14-40 to 14-41

1. Seasonal variations, nonconformity of the environments, effects of other interactions, unexpected disasters, etc.
3. This model assumes the number of interactions is proportional to the product of x and y:

$$\frac{dx}{dt} = (a - by)x, \quad a < 0,$$

$$\frac{dy}{dt} = m\left(1 - \frac{y}{M}\right)y - nxy = y\left(m - \frac{m}{M}y - nx\right).$$

Rest points are (0, 0), unstable, and (0, M), stable.

5. **(a)** Logistic growth occurs in the absence of the competitor, and involves a simple interaction between the species: growth dominates the competition when either population is small, so it is difficult to drive either species to extinction.

(b) a: per capita growth rate for trout
m: per capita growth rate for bass
b: intensity of competition to the trout
n: intensity of competition to the bass
k_1: environmental carrying capacity for the trout
k_2: environmental carrying capacity for the bass
$\frac{a}{b}$: growth versus competition or net growth of trout
$\frac{m}{n}$: relative survival of bass

(c) $\frac{dx}{dt} = 0$ when $x = 0$ or $y = \frac{a}{b} - \frac{a}{bk_1}x$,
$\frac{dy}{dt} = 0$ when $y = 0$ or $y = k_2 - \frac{k_2 n}{m}x$.

By picking $a/b > k_2$ and $m/n > k_1$, we insure that an equilibrium point exists inside the first quadrant.

CHAPTER 15

Section 15.1, p. 15-7

1. $y = c_1e^{-3x} + c_2e^{4x}$ **3.** $y = c_1e^{-4x} + c_2e^{x}$
5. $y = c_1e^{-2x} + c_2e^{2x}$ **7.** $y = c_1e^{-x} + c_2e^{3x/2}$
9. $y = c_1e^{-x/4} + c_2e^{3x/2}$ **11.** $y = c_1\cos 3x + c_2\sin 3x$
13. $y = c_1\cos 5x + c_2\sin 5x$ **15.** $y = e^{x}(c_1\cos 2x + c_2\sin 2x)$
17. $y = e^{-x}\left(c_1\cos\sqrt{3}x + c_2\sin\sqrt{3}x\right)$
19. $y = e^{-2x}\left(c_1\cos\sqrt{5}x + c_2\sin\sqrt{5}x\right)$
21. $y = c_1 + c_2x$ **23.** $y = c_1e^{-2x} + c_2xe^{-2x}$
25. $y = c_1e^{-3x} + c_2xe^{-3x}$ **27.** $y = c_1e^{-x/2} + c_2xe^{-x/2}$
29. $y = c_1e^{-x/3} + c_2xe^{-x/3}$ **31.** $y = -\frac{3}{4}e^{-5x} + \frac{3}{4}e^{-x}$
33. $y = \frac{1}{2\sqrt{3}}\sin 2\sqrt{3}x$
35. $y = -\cos 2\sqrt{2}x + \frac{1}{\sqrt{2}}\sin 2\sqrt{2}x$
37. $y = (1 - 2x)e^{2x}$ **39.** $y = 2(1 + 2x)e^{-3x/2}$
41. $y = c_1e^{-x} + c_2e^{3x}$ **43.** $y = c_1e^{-x/2} + c_2xe^{-x/2}$
45. $y = c_1\cos\sqrt{5}x + c_2\sin\sqrt{5}x$ **47.** $y = c_1e^{-x/5} + c_2xe^{-x/5}$
49. $y = e^{-x/2}(c_1\cos x + c_2\sin x)$ **51.** $y = c_1e^{3x/4} + c_2xe^{3x/4}$
53. $y = c_1e^{-4x/3} + c_2xe^{-4x/3}$ **55.** $y = c_1e^{-x/2} + c_2e^{4x/3}$
57. $y = (1 + 2x)e^{-x}$ **59.** $y = \frac{15}{13}e^{-7x/3} + \frac{11}{13}e^{2x}$

Section 15.2, p. 15-16

1. $y = c_1e^{5x} + c_2e^{-2x} + \frac{3}{10}$
3. $y = c_1 + c_2e^{x} + \frac{1}{2}\cos x - \frac{1}{2}\sin x$
5. $y = c_1\cos x + c_2\sin x - \frac{1}{8}\cos 3x$
7. $y = c_1e^{2x} + c_2e^{-x} - 6\cos x - 2\sin x$
9. $y = c_1e^{x} + c_2e^{-x} - x^2 - 2 + \frac{1}{2}xe^{x}$
11. $y = c_1e^{3x} + c_2e^{-2x} - \frac{1}{4}e^{-x} + \frac{49}{50}\cos x + \frac{7}{50}\sin x$
13. $y = c_1 + c_2e^{-5x} + x^3 + \frac{3}{5}x^2 - \frac{6}{25}x$
15. $y = c_1 + c_2e^{3x} + 2x^2 + \frac{4}{3}x + \frac{1}{3}xe^{3x}$
17. $y = c_1 + c_2e^{-x} + \frac{1}{2}x^2 - x$
19. $y = c_1\cos x + c_2\sin x - \frac{1}{2}x\cos x$
21. $y = (c_1 + c_2x)e^{-x} + \frac{1}{2}x^2e^{-x}$
23. $y = c_1e^{x} + c_2e^{-x} + \frac{1}{2}xe^{x}$
25. $y = e^{-2x}(c_1\cos x + c_2\sin x) + 2$
27. $y = A\cos x + B\sin x + x\sin x + \cos x\ln(\cos x)$
29. $y = c_1 + c_2e^{5x} + \frac{1}{10}x^2e^{5x} - \frac{1}{25}xe^{5x}$
31. $y = c_1\cos x + c_2\sin x - \frac{1}{2}x\cos x + x\sin x$
33. $y = c_1 + c_2e^{x} + \frac{1}{2}e^{-x} + xe^{x}$
35. $y = c_1e^{5x} + c_2e^{-x} - \frac{1}{8}e^{x} - \frac{4}{5}$
37. $y = c_1\cos x + c_2\sin x - (\sin x)[\ln(\csc x + \cot x)]$
39. $y = c_1 + c_2e^{8x} + \frac{1}{8}xe^{8x}$
41. $y = c_1 + c_2e^{x} - x^4/4 - x^3 - 3x^2 - 6x$
43. $y = c_1 + c_2e^{-2x} - \frac{1}{3}e^{x} + x^3/6 - x^2/4 + x/4$
45. $y = c_1\cos x + c_2\sin x + (x - \tan x)\cos x - \sin x\ln(\cos x)$
$= c_1\cos x + c_2'\sin x + x\cos x - (\sin x)\ln(\cos x)$
47. $y = ce^{3x} - \frac{1}{2}e^{x}$

49. $y = ce^{3x} + 5xe^{3x}$

51. $y = 2\cos x + \sin x - 1 + \sin x \ln(\sec x + \tan x)$

53. $y = -e^{-x} + 1 + \frac{1}{2}x^2 - x$

55. $y = 2(e^x - e^{-x})\cos x - 3e^{-x}\sin x$

57. $y = (1 - x + x^2)e^x$

59. $y_p = \frac{1}{4}x^2$

Section 15.3, pp. 15-21 to 15-23

1. $my'' + y' + y = 0, \quad y(0) = 2, \ y'(0) = 2$

3. $\frac{25}{32}y'' + 40y = 0, \quad y(0) = \frac{5}{12}, \ y'(0) = \frac{v_0}{12}$

5. $2q'' + 4q' + 10q = 20\cos t, \quad q(0) = 2, \ q'(0) = 3$

7. 0.0864 ft (above equilibrium)

9. $y(t) = 0.2917\cos(7.1552t) + \frac{v_0}{85.8623}\sin(7.1552t)$ (in feet), or $y = 3.5\cos(7.1552t) + \frac{v_0}{0.1398}\sin(7.1552t)$ (in inches).

11. 0.308 sec **13.** 8.334 lb **15.** 24.4949 ft/sec

17. -1.56 ft/sec^2 (acceleration upward)

19. $q(t) = -8e^{-3t} + 10e^{-2t}, \quad \lim_{t\to\infty} q(t) = 0$

21. $y(t) = 1 + 2e^{-t} - \frac{1}{3}e^{-2t} - \frac{2}{3}e^{-8t}$

23. $y(\pi) = -2$ m (above equilibrium)

25. $q(t) = \frac{1}{5} + \left(\frac{49\sqrt{199}}{995}\sin\frac{\sqrt{199}}{2}t + \frac{49}{5}\cos\frac{\sqrt{199}}{2}t\right)e^{-t/2}$

Section 15.4, p.15-25

1. $y = \frac{c_1}{x^2} + c_2 x$ **3.** $y = \frac{c_1}{x^2} + c_2 x^3$

5. $y = c_1 x^2 + c_2 x^4$ **7.** $y = c_1 x^{-1/3} + c_2$

9. $y = x(c_1 + c_2 \ln x)$

11. $y = x[c_1\cos(2\ln x) + c_2\sin(2\ln x)]$

13. $y = \frac{1}{x}[c_1\cos(3\ln x) + c_2\sin(3\ln x)]$

15. $y = \frac{1}{\sqrt{x}}[c_1\cos(\ln x) + c_2\sin(\ln x)]$

17. $y = \frac{1}{x}(c_1 + c_2\ln x)$ **19.** $y = c_1 + c_2\ln x$

21. $y = \frac{1}{\sqrt[3]{x}}(c_1 + c_2\ln x)$ **23.** $y = x^{-5/4}(c_1 + c_2\ln x)$

25. $y = \frac{1}{2x^3} + \frac{x}{2}$ **27.** $y = x$

29. $y = x[-\cos(\ln x) + 2\sin(\ln x)]$

Section 15.5, p. 15-31

1. $y = c_0 + c_1\left(x - x^2 + \frac{2}{3}x^3 - \cdots\right)$
$= c_0 - \frac{c_1}{2}e^{-2x}$

3. $y = c_0(1 - 2x^2 + \cdots) + c_1\left(x - \frac{2}{3}x^3 + \cdots\right)$
$= c_0\cos 2x + c_1\sin 2x$

5. $y = c_1 x + c_2 x^2$

7. $y = c_0\left(1 + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \cdots\right) + c_1\left(x + \frac{1}{6}x^3 + \cdots\right)$

9. $y = c_0\left(1 - x^2 + \frac{5}{12}x^4 - \cdots\right) + c_1 x$

11. $y = c_0(1 - 3x^2 + \cdots) + c_1(x - x^3)$

13. $y = c_0\left(1 + x^2 + \frac{2}{3}x^4 + \cdots\right)$
$+ c_1\left(x + x^3 + \frac{3}{5}x^5 + \cdots\right)$

15. $y = c_0\left(1 - \frac{3}{2}x^2 + \cdots\right) + c_1\left(x - \frac{1}{2}x^3 + \cdots\right)$

17. $y = c_0\left(1 - \frac{3}{2}x^2 + \frac{1}{8}x^4 + \cdots\right) + c_1\left(x - \frac{1}{3}x^3\right)$