

PROBLEM 1A _____

Do there exist 10 natural numbers such that none one of them is divisible by another one, and the square of any one of them is divisible by any other of the original numbers?

PROBLEM 2A _____

Consider 101 natural numbers not exceeding 200. Prove that at least one of them is divisible by another one.

PROBLEM 3A _____

Given a triple of numbers one is allowed to perform the following operation. Take any two of them, say a and b out, and put $(a + b)/\sqrt{2}$ and $(a - b)/\sqrt{2}$ instead. Is it possible to produce the triple $(1, \sqrt{2}, 1 + \sqrt{2})$ out of $(2, \sqrt{2}, 1/\sqrt{2})$ after a number of iterations of the allowed operation?

PROBLEM 4A _____

The sequence $1, 11, 111, 1111, 11111, \dots$ does not contain numbers divisible by 2005 and 2006. Indeed, the former number is divisible by 5, and the latter number is divisible by 2, whereas neither member of the sequence is divisible by 2 or 5. Does this sequence contain any member divisible by 2007?

PROBLEM 5A _____

There are n points in the plane. All the midpoints of all segments which have given points as their endpoints are marked. Prove that at least $2n - 3$ points are marked.

PROBLEM 1B _____

A country has a hundred airports. All distances between the pairs of airports are different. One day planes starting in every airport head to the closest airport. Assume that one and only one plane flies from each airport. What is the maximal number of planes landing this day at the same airport?

PROBLEM 2B _____

Consider a hundred of integers. Prove that one can pick several of them (maybe only one) such that their sum is divisible by 100.

PROBLEM 3B _____

Several positive numbers are arranged into a rectangular array. The product of the sum of numbers in any column with the sum of the numbers in any row equals the number which occupies the intersection of this column and this row. Find the sum of all numbers in the array.

PROBLEM 4B _____

There are $2k + 1$ cards with the numbers $1, 2, 3, \dots, 2k + 1$ written on them. What is the maximal number of cards which one can pick such that no one chosen number equals the sum of two other chosen numbers?

PROBLEM 5B _____

A finite set of points is situated in the plane in such a way that every straight line through any two of them passes through at least one more point from this set. Prove that all the points belong to one straight line.