Question 1

(This is exercise 10 on p. 386) Let $K$ be a Galois extension of $F$. Two intermediate fields $E$ and $L$ are said to be conjugate if there exists $\sigma \in \text{Gal}_F K$ such that $\sigma(E) = L$. Prove that $E$ and $L$ are conjugate if and only if $\text{Gal}_E K$ and $\text{Gal}_L K$ are conjugate subgroups of $\text{Gal}_F K$. 
Question 2

Provide an example of a field extension $K \supset \mathbb{Q}$ which is a radical extension and not a normal extension. Prove these properties of your example.
Question 3

Does there exist an irreducible polynomial $p \in \mathbb{Q}[x]$ with multiple roots? Verify your answer.
Let $K = \mathbb{Q}(\sqrt{2}, \sqrt{-2})$.

a) Extend the set \{\sqrt{2}, \sqrt{-2}\} to a basis of $K$ as a vector field over $\mathbb{Q}$.

b) Find $\text{Gal}_{\mathbb{Q}}K$. 
Question 5

Let $p \in \mathbb{Q}[x]$ be an irreducible polynomial. Suppose $K$ is an extension field of $\mathbb{Q}$ that contains a root $\alpha$ of $p$ such that $p(\alpha^2) = 0$. (That is $\alpha^2$ is also a root of $p$.) Prove that $p$ splits in $K[x]$. 