Name: ______________________________

Question 1

(This is exercise 10 on p. 386) Let K be a Galois extension of F. Two intermediate fields E and L are said to be conjugate if there exists $\sigma \in \mathrm{Gal}_F K$ such that $\sigma(E) = L$. Prove that E and L are conjugate if and only if $\mathrm{Gal}_E K$ and $\mathrm{Gal}_L K$ are conjugate subgroups of $\mathrm{Gal}_F K$.

Question 2

Provide an example of a field extension $K \supset \mathbb{Q}$ which is a radical extension and not a normal extension. Prove these properties of your example.

Question 3

Does there exist an irreducible polynomial $p \in \mathbb{Q}[x]$ with multiple roots? Verify your answer.

Question 4

Let $K = \mathbb{Q}(\sqrt{2}, \sqrt{-2})$.

a) Extend the set $\{\sqrt{2}, \sqrt{-2}\}$ to a basis of K as a vector field over $\mathbb{Q}$.

b) Find $\mathrm{G}al_{\mathbb{Q}}K$.

Question 5

Let $p \in \mathbb{Q}[x]$ be an irreducible polynomial. Suppose K is an extension field of $\mathbb{Q}$ that contains a root α of p such that $p(\alpha^2) = 0$. (That is α^2 is also a root of p.) Prove that p splits in $K[x]$.