On p-adic families of Siegel cusp forms in the Maaß Spezialschar

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0. Introduction

p-adic families of Siegel modular forms were considered from various points of view in recent years [10], [7]. In particular, p-adic families of Eisenstein series were considered by A. Panchishkin and K.Kitagawa. In this connection, the question of whether one can assemble Siegel cusp eigenforms into p-adic families (from the point of view of their Fourier coefficients) becomes interesting. The nature of the Fourier coefficients of arbitrary Siegel cusp forms remains vague even in the case of degree two (see [8] for numerical examples and discussion). In general, when the degree is greater than one, the theory of Fourier coefficients and the theory of Hecke eigenvalues are not "proportional" at all, and so the situation is quite different from the one-variable case. The picture, however, becomes much clearer if one considers the Siegel cusp forms which are connected to elliptic cusp forms via a lifting procedure.

Consider the Maaß lifting, as described in [11], [2]. One should mention that forms in the Maaß space are of course strictly speaking not "true" Siegel modular forms, in the sense that they are obtained by means of a lifting process from GL_2. Thus many properties can be derived from corresponding properties of forms on GL_2. In this case, the squares of Fourier coefficients of the cusp Siegel eigenform are essentially equal to the central special values of the L-function associated to the elliptic cusp form. Thus, informally speaking, the "square" of a p-adic interpolation result follows from "restriction to diagonal" of two-variable p-adic L-functions. Meanwhile, in order to obtain the result on p-adic interpolation of the Fourier coefficients themselves, instead of their squares, we need a more delicate technique.

Consider a normalized p-ordinary Λ-adic cusp eigenform of tame conductor 1. Its specializations at certain arithmetic points are p-stabilized newforms, which are actually oldforms of level p and trivial Nebentypus. Consider the corresponding newforms on $SL_2(\mathbb{Z})$, and apply the Maaß lifting as in [2], [11]. We get in this way an infinite collection of Siegel cusp forms. Thus, the p-adic interpolation problem for their Fourier coefficients makes sense. We describe this procedure and formulate our result in the first section of the paper. The additional Euler factors in our theorem mirror the difference between the p-stabilized newforms and the complex-analytic newforms on $SL_2(\mathbb{Z})$.

We introduce Jacobi forms in the second section, recall their place in the construction of Maaß lifting [2], and the description of their Fourier coefficients in terms of cycle integrals [4]. Note that a p-adic family of Jacobi forms was considered in [6]. This family corresponds to a p-ordinary Λ-adic form of non-trivial tame level. The form comes from powers of a Hecke character. A certain conjecture was posed in loc. cit. The technique of the present paper is applicable in this setting as well. The corresponding paper will be published elsewhere.

The fourth section is devoted to the proof of the theorem of section 1. Here we make use of the p-adic interpolation of cycle integrals from [9]. Not all the cycle integrals may be interpolated p-adically, in fact only those associated with quadratic forms satisfying certain congruence conditions. For this reason, we have to show that one can assemble everything, making use of only these quadratic forms. This is the subject of the third section of the paper.

The question about the Λ-adic Maaß lifting was posed by E. Freitag on a conference which took place at Institut Fourier (Grenoble), 1996. This work was undertaken to answer this question. I wish to express deep gratitude to A.Panchishkin who transmitted the question to me. His long-term interest in the subject gave me much encouragement.

1. Notations and statement of the theorem.

A Siegel modular form of degree two F has the Fourier expansion

$$F(\tau, z, \tau') = \sum_{\substack{n,r,m\in\mathbb{Z}\\ n,m,4mn-r^2\geq 0}} A(n,r,m)e(n\tau + rz + m\tau').$$

The Maaß Spezialschar contains the forms satisfying the condition

$$A(n,r,m) = \sum_{d|(n,r,m)} d^k A(nm/d^2, r/d, 1). \tag{1}$$

The Hecke algebra for the Siegel modular forms is generated by certain operators $T_S(l)$ and $T_S(l^2)$ with prime numbers l (we follow the notations of [2]). Given an eigenform F in the Maaß Spezialschar, set

$$F|T_S(l) = \gamma_l F \quad \text{and} \quad F|T'_S(l) = \gamma'_l F,$$

where $T'_S(l) = T_S(p)^2 - T_S(p^2)$. The associated zeta-function is defined by the Euler product

$$Z_F(s) = \prod_l (1 - \gamma_l l^{-s} + (\gamma'_l - l^{2k-2})l^{-2s} - \gamma_l l^{2k-1-3s} + l^{4k-2-4s})^{-1}.$$

The previous Saito-Kurokawa conjecture, proven by Maaß Andrianov and Zagier, ([2], [11]), establishes an isomorphism, as modules over the Hecke algebra, between the Maaß Spezialschar of weight $k+1$ and the space of elliptic cusp forms of weight $2k$. If f is an elliptic cusp Hecke eigenform of weight $2k$, and $L(f,s)$ is its L-function, one has

$$Z_F(s) = \zeta(s-k+1)\zeta(s-k+2)L(f,s).$$

In this paper, our object of consideration is the set of numbers $A(n,r,m)$, where the corresponding normalized elliptic cusp Hecke eigenform $f_{2k} = \sum_{n>0} a_{2k}(n)q^n$ comes from a p-adic family.

Evidently, the numbers $A(n,r,m)$ are defined up to a common non-zero multiple.

The construction [2] yields that $A(nm,r,1)$ depends only on $r^2 - 4nm$.

Put $B(N) = A(nm,r,1)$ for $N = 4nm - r^2$. Let $D < 0$ be a fundamental discriminant (i.e. a discriminant of a quadratic field). Then

$$B(t^2|D|) = B(|D|)\sum_{d|t} \mu(d)\left(\frac{D}{d}\right) d^k a_{2k}(t/d). \tag{2}$$

Let us now vary the elliptic cusp Hecke eigenform. Let $p > 5$ be a rational prime. Fix an embedding $\bar{\mathbb{Q}} \hookrightarrow \hat{\bar{\mathbb{Q}}}_p$ of algebraic numbers into Tate's field. In the following, we will not distinguish between algebraic numbers and their images under this embedding. Let $\Lambda = \mathbb{Z}_p[[1 + p\mathbb{Z}_p]]$ be the completed group ring on the principal unit group $1 + p\mathbb{Z}_p$. Put $\Lambda_1 = \Lambda[(\mathbb{Z}/p\mathbb{Z})^*]$. Let $\mathcal{R}_1$ be the universal p-ordinary Hecke algebra of tame conductor 1 [9]. For a finite flat Λ-algebra $\mathcal{R}$ we put

$$\mathcal{X}(\mathcal{R}) = Hom_{cont}(\mathcal{R}, \overline{\mathbb{Q}}_p).$$

Let k be an odd integer, and let f_{2k} be a normalized p-ordinary cusp Hecke eigenform of weight $2k$ on $SL_2(\mathbb{Z})$. Put

$$f^*_{2k}(\tau) = f_{2k}(\tau) - \beta_{2k} f_{2k}(p\tau), \tag{3}$$

where $\alpha_{2k}\beta_{2k} = p^{2k-1}$, the eigenvalue of the p-th Hecke operator $a_{2k}(p) = \alpha_{2k} + \beta_{2k}$, and $|\alpha_{2k}|_p = 1$. Thus f^*_{2k} is an ordinary p-stabilized newform. Pick the arithmetic point κ of signature $(2k, \omega^{2k})$ corresponding to f^*_{2k}. The symbol ω denotes the Teichmüller character. We have the natural finite-to-one mapping

$$\pi : \ \mathcal{X}(\mathcal{R}_1) \to \mathcal{X}(\Lambda_1) \to \mathcal{X}(\Lambda).$$

Since κ is arithmetic, it is unramified over $\mathcal{X}(\Lambda)$ by a theorem of Hida ([3], Theorem 2.5 b). Thus, there is a neighborhood U of $\pi(\kappa)$ in $\mathcal{X}(\Lambda)$, and a local section of π in U (see [9], p.132 for the construction of this section. We get so far a family of ordinary p-stabilized newforms parameterized by points of U. We embed $\mathbb{Z}_p \times \mathbb{Z}/(p-1)\mathbb{Z}$ into $\mathcal{X}(\Lambda_1)$ identifying a pair (l, u) with the arithmetic point of signature (l, ω^u). We identify an even integer $2k$ with $(2k, 2k \bmod (p-1)) \in \mathbb{Z}_p \times \mathbb{Z}/(p-1)\mathbb{Z}$. The corresponding p-stabilized newform f^*_{2k} has trivial Nebentypus and level p. Thus, by (3), we obtain the cusp Hecke eigenform f_{2k} of weight $2k$ on $SL_2(\mathbb{Z})$. Pick k_0 such that $(k_0, k_0 \bmod p-1) \in U$. Thus we obtain the family $\{f_{2k}\}$ parameterized by weights with $2k \equiv 2k_0 \bmod (p-1)p^N$ with sufficiently large N. This is not a p-adic analytic family in the sense of [5] (though $\{f^*_{2k}\}$ is).

Denote by

$$F_{2k} = \sum_{\substack{n,r,m \in \mathbb{Z} \\ n,m,4nm-r^2>0}} A_{2k}(n,r,m) \exp(2\pi i(n\tau + rz + m\tau'))$$

the cusp Siegel modular form of weight $k+1$ in the Maaß Spezialschar corresponding to f_{2k} as above.

Now we can formulate our main result.

Theorem. *Assume that there exist fundamental discriminants* $D_0 = r_0^2 - 4n_0m_0 \equiv 0 \bmod p$ *and* $D_1 = r_1^2 - 4n_1m_1 \not\equiv 0 \bmod p$ *such that*

$$A_{2k_0}(n_0m_0, r_0, 1) \neq 0 \tag{4}$$

and

$$A_{2k_0}(n_1m_1, r_1, 1) \neq 0 \tag{5}$$

Then there exist a normalization of the Siegel modular forms F_{2k}, and analytic functions $\mathcal{A}_D$ on U such that

$$\mathcal{A}_D(\kappa) = \left(1 - \left(\frac{D}{p}\right)\beta_{2k}p^{-k}\right) A_{2k}(n,r,m)$$

for any fundamental discriminant $D = r^2 - 4nm$, and $\kappa \in U$.

Remarks. 1. Together with (1) and (2), this provides the description of p-adic behavior of all the Fourier coefficients of the Siegel cusp forms in the family.

2. We do not consider Eisenstein series here. However, one can formally look at them as "Maaß lifting" of elliptic Eisenstein series on $SL_2(\mathbb{Z})$. The theorem remains true for this case. This is just because the numbers $A_{2k}(n,r,m)$ are equal in this case to Cohen's generalized class numbers [1]:

$$A_{2k}(n,r,m) = H(k, r^2 - 4nm)$$

These are special values of the Dirichlet L-function at negative integers, and, therefore, the assertion follows from the construction of Kubota - Leopoldt p-adic L-function.

Much more general results in this direction for Eisenstein series (of arbitrary degree, and twisted with a cyclotomic character) were recently obtained by A.Panchishkin [7].

3. The author does not know a situation when the assumptions (4) and (5) do not hold. I can not however, get rid of (4). As to (5), it is included just in order not to overload the text with the proof that it always holds.

2. Jacobi forms.

The space J_{k+1}^{cusp} of Jacobi forms of weight $k+1$ is isomorphic as a module over the Hecke algebra to the space of elliptic cusp forms of weight $2k$ [2]. Let

$$\phi_{k+1}(\tau, z) = \sum_{n>0,\ r^2<4n} c_{2k}(n,r) q^n \zeta^r \qquad q = \exp(2\pi i \tau), \qquad \zeta = \exp(2\pi i z)$$

be the Jacobi eigenform of weight $k+1$ which corresponds to f_{2k}. The numbers $c_{2k}(n,r)$ are defined up to a common non-zero multiple. According to [2], we may put

$$A_{2k}(n,r,1) = c_{2k}(n,r). \tag{6}$$

This, together with (1) defines the numbers $A_{2k}(n,r,m)$ in question. Thus we will prove the theorem as a statement on Fourier coefficients of Jacobi forms rather than on the Fourier coefficients of Siegel cusp forms in Maaß Spezialschar.

The numbers $c_{2k}(n,r)$ now become the object of our attention. For the sake of simplifying the notations, we will sometimes drop the index $2k$.

In order to recall the description of these numbers in terms of cycle integrals, we need some additional notations. We consider integral binary quadratic forms

$$Q(x,y) = [a,b,c](x,y) = ax^2 + bxy + cy^2.$$

The group action of $SL_2(\mathbb{Z})$

$$[a,b,c] \circ \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = [a,b,c](\alpha x + \beta y, \gamma x + \delta y)$$

preserves the discriminant $\Delta = b^2 - 4ac$ and the greatest common divisor (a, b, c). The number of classes is finite. We denote by $\mathcal{Q}_\Delta$ the set of all quadratic forms with discriminant Δ. For a fundamental discriminant D_0 dividing Δ, denote by $\chi_{D_0} : \mathcal{Q}_{D_0} \to \{\pm 1, 0\}$ the generalized genus character as in [4].

The cycle integral is defined by ([4])

$$r_{k,Q}(f_{2k}) = \int_{C_Q} f(\tau) Q(\tau, 1)^{k-1} d\tau$$

where C_Q is the image in $SL_2(\mathbb{Z})\backslash\mathfrak{H}$ of the semicircle $a|\tau|^2 + b\Re\tau + c = 0$ (oriented from left to right if $a > 0$, from right to left if $a < 0$, and from $-c/b$ to $i\infty$ if $a = 0$.

Specializing [4], Section II.4, to our setting, we get the following.

Proposition. *Let $D_0 = r_0^2 - 4n_0 < 0$ be a fundamental discriminant. Then for all $n, r \in \mathbb{Z}$ with $D = r^2 - 4n < 0$, we have*

$$c(n, r)c(n_0, r_0) = C_k \sum_{Q \in \mathcal{Q}_{DD_0}/SL_2(\mathbb{Z})} \chi_{D_0}(Q) r_{k,Q}(f_{2k}) \tag{7}$$

The number C_k does not depend on n, r, n_0, r_0.

The sum on the right in (7) does not depend on the particular choice of representatives $Q \in \mathcal{Q}_{DD_0}/SL_2(\mathbb{Z})$. We make certain special choices below.

3. Preparation Lemmas.

Lemma 1. *Assume $\Delta \equiv 0 \bmod p$. Then one can choose a representative system $Q = [a, b, c] \in \mathcal{Q}_\Delta/SL_2(\mathbb{Z})$ such that $a \equiv b \equiv 0 \bmod p$ for any Q.*

Proof. Indeed, for a quadratic form $Q = [a, b, c]$ with $b \not\equiv 0 \bmod p$ we have $a \not\equiv 0 \bmod p$ (otherwise $\Delta = b^2 - 4ac \not\equiv 0 \bmod p$). Choose $\beta \equiv -b/(2a) \bmod p$. Thus $Q \circ \begin{pmatrix} 1 & \beta \\ 0 & 1 \end{pmatrix}$ is $SL_2(\mathbb{Z})$-equivalent to Q and satisfies $b \equiv 0 \bmod p$. Therefore one can assume $b \equiv 0 \bmod p$. If $a \not\equiv 0 \bmod p$ then $c \equiv 0 \bmod p$ (because $\Delta \equiv 0 \bmod p$). Thus $Q \circ \begin{pmatrix} p & p-1 \\ 1 & 1 \end{pmatrix}$ lies in the same class as Q and satisfies $a \equiv b \equiv 0 \bmod p$ as desired.

We will make use of the following property of the system of representatives.

Lemma 2. *Assume $\Delta \equiv 0 \bmod p$ and $\Delta \not\equiv 0 \bmod p^2$. If $\{Q = [a, b, c]\}$ with $a \equiv b \equiv 0 \bmod p$ is a representative system of $\mathcal{Q}_\Delta/SL_2(\mathbb{Z})$, then $\{Q = [a/p, b, cp]\}$ is also a representative system.*

Proof. Since the number of classes is finite, it is sufficient to show that if $[a/p, b, cp]$ is $SL_2(\mathbb{Z})$-equivalent to $[a'/p, b', c'p]$, then $[a, b, c]$ is $SL_2(\mathbb{Z})$-equivalent to $[a', b', c']$. Assume $[a/p, b, pc] \circ \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = [a'/p, b', pc']$. Since $\Delta \not\equiv 0 \bmod p^2$, we have $a \not\equiv 0 \bmod p^2$. Thus $pc' = (a/p)\beta^2 + b\beta\delta + pc\delta^2$ yields $\beta \equiv 0 \bmod p$. It follows that $[a, b, c] \circ \begin{pmatrix} \alpha & \beta/p \\ \gamma p & \delta \end{pmatrix} = [a', b', c']$, as desired.

Notice that, under the assumption of Lemma 2, if $D_0 \equiv 0 \bmod p$,

$$\chi_{D_0}([a/p, b, pc]) = \left(\frac{\Delta/D_0}{p}\right) \chi_{D_0}([a, b, c]). \tag{8}$$

This easily follows from [4], Proposition 1, p.508.

Choose the system of representatives as in Lemma 1. We get for a fundamental discriminant $D_0 \equiv 0 \bmod p$ and a discriminant $D \not\equiv 0 \bmod p$

$$\sum_{\mathcal{Q}_{DD_0}/SL_2(\mathbb{Z})} \chi_{D_0}(Q) r_{k,Q}(f_{2k}^*) = \sum_{\mathcal{Q}_{DD_0}/SL_2(\mathbb{Z})} \chi_{D_0}(Q) r_{k,Q}(f_{2k}(\tau) - \beta_{2k} f_{2k}(p\tau)) =$$
$$\left(1 - \left(\frac{D}{p}\right) \beta_{2k} p^{-k}\right) \sum_{\mathcal{Q}_{DD_0}/SL_2(\mathbb{Z})} \chi_{D_0}(Q) r_{k,Q}(f_{2k}). \tag{9}$$

The last equality follows from Lemma 2 and (8) after the variable change $\tau \mapsto \tau/p$ in the second term.

Assume now that the discriminant $D \equiv 0 \bmod p$, and $D \not\equiv 0 \bmod p^2$. We claim that for a fundamental discriminant $D_0 \equiv 0 \bmod p$

$$\sum_{Q \in \mathcal{Q}_{DD_0}/SL_2(\mathbb{Z})} \chi_{D_0}(Q) r_{k,Q}(f_{2k}^*) = \sum_{Q \in \mathcal{Q}_{DD_0}/SL_2(\mathbb{Z})} \chi_{D_0}(Q) r_{k,Q}(f_{2k}) \tag{10}$$

at least under the assumption

$$\left(\frac{DD_0/p^2}{p}\right) = 1. \tag{11}$$

Indeed, in the view of (3), it is sufficient to show that

$$\sum_{Q \in \mathcal{Q}_{DD_0}/SL_2(\mathbb{Z})} \chi_{D_0}(Q) \int_{C_Q} f_{2k}(p\tau)(a\tau^2 + b\tau + c)^{k-1} d\tau = 0 \tag{12}$$

for a special choice of representative system $Q \in \mathcal{Q}_{DD_0}/SL_2(\mathbb{Z})$.

Lemma 3. *Let $D_0 \equiv 0 \bmod p$ be a fundamental discriminant and D be a discriminant such that $(D, p^2) = p$. Assume (11). Then one can pick a representative system $Q = [a, b, c] \in \mathcal{Q}_{DD_0}/SL_2(\mathbb{Z})$ such that $a \equiv 0 \bmod p^3$ and $b \equiv 0 \bmod p$ unless $(a, b, c) \equiv 0 \bmod p$.*

Proof. Lemma 1 yields that we can assume $a \equiv 0 \bmod p^2$ and $b \equiv 0 \bmod p$. A quadratic form $[a, b, c]$ is $SL_2(\mathbb{Z})$-equivalent to $[a', b', c']$ with

$$a' = a\alpha^2 + b\beta\gamma + c\gamma^2$$
$$b' = 2a\alpha\beta + b\alpha\delta + b\beta\gamma + 2c\gamma\delta$$
$$c' = a\beta^2 + b\beta\delta + c\delta^2$$

for any $\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \in SL_2(\mathbb{Z})$. Put $\gamma \equiv 0 \bmod p$. It follows from the assumption (11) that the quadratic form in $(\alpha, \gamma/p)$

$$\frac{a}{p^2}\alpha^2 + \frac{b}{p}\alpha\frac{\gamma}{p} + c\frac{\gamma}{p}^2$$

represents zero modulo p. The assertion of Lemma 3 follows.

Now we notice that the integrals in (12) are stable under the variable change $\tau \mapsto \tau + 1/p$. It follows that the sum in the left in (12) splits into the sums

$$\sum_{\alpha \bmod p} \chi_{D_0}([a, 2\frac{a}{p}\alpha + b, \frac{a}{p^2}\alpha^2 + \frac{b}{p}\alpha + c])$$

multiplied by integrals $\int_{C_Q} f_{2k}(p\tau)(a\tau^2 + b\tau + c)^{k-1} d\tau$ for $Q \in [a, b, c]$. Since

$$\chi_{D_0}([a, 2\frac{a}{p}\alpha + b, \frac{a}{p^2}\alpha^2 + \frac{b}{p}\alpha + c]) = \left(\frac{D_0/p}{a}\right)\left(\frac{p}{\alpha^2 a/p^2 + \alpha b/p + c}\right), \tag{13}$$

$\alpha^2 a/p^2 + \alpha b/p + c \equiv \alpha b/p + c$, and $\sum_{\alpha \bmod p}\left(\frac{p}{\alpha b/p + c}\right) = 0$, the sum (13) vanishes. This proves (12).

4. Proof of the theorem.

For a quadratic form $Q = [a, b, c]$ we put $Q^\iota = [a, -b, c]$. Our p-adic interpolation argument is essentially based on the following result of Stevens. The statement below specializes to our setting a special case of Theorem 5.5 (see also Lemma 6.1) of [9].

Proposition. *There exist*

- *complex numbers $\Omega^-(\kappa) \neq 0$ for $\kappa \in U$,*
- *p-adic periods $\Omega_\kappa \in \kappa(\Lambda)$ for $\kappa \in U$ with $\Omega_{\kappa_0} \neq 0$*
- *p-adic analytic $\overline{\mathbb{Q}}_p$-valued functions $J_Q(\kappa)$ defined for any $Q = [a, b, c]$ with $a \equiv b \equiv 0 \bmod p$ on $\kappa \in U$ with the following property.*

If $\kappa \in U$ is an arithmetic point lying under $(2k, 2k_0 \bmod p - 1) \in \mathcal{X}(\Lambda)$ with $2k \equiv 2k_0 \bmod p - 1$, then

$$J_Q(\kappa) = \frac{\Omega_\kappa}{\Omega^-(\kappa)}\left(r_{k,Q}(f_{2k}^*) + r_{k,Q^\iota}(f_{2k}^*)\right).$$

Both sides of the identity are algebraic numbers.

Notice that when Q runs through a representative system for $\mathcal{Q}_{DD_0}/SL_2(\mathbb{Z})$, the form Q^ι also runs through a representative system for $\mathcal{Q}_{DD_0}/SL_2(\mathbb{Z})$.

Putting now together (6), (7), (9), Lemma 1 and the above Proposition, we finish the proof of our theorem in the case when

$$D = r^2 - 4nm \not\equiv 0 \bmod p$$

and, with (10) instead of (9), in the case when

$$D = r^2 - 4nm = 0 \bmod p \quad \text{with} \quad \left(\frac{DD_0/p^2}{p}\right) = 1.$$

The number $c_{2k}(n_0, r_0)$, which is assumed to be non-zero at $k = k_0$, becomes a part of a normalization factor.

It is somehow astonishing, but the author is not able to produce a proof along the same lines in the case when

$$D = r^2 - 4nm = 0 \bmod p \quad \text{with} \quad \left(\frac{DD_0/p^2}{p}\right) = -1.$$

This situation is similar to that which one encounters in [6].

Fortunately, we have the following alternate route. Assume $D_2 = r_2^2 - 4n_2 \equiv 0 \bmod p$ is such that

$$\left(\frac{D_2 D_0/p^2}{p}\right) = -1.$$

Then, for $c(n_1, r_1) \neq 0$,

$$c(n_0, r_0)c(n_2, r_2) = \frac{c(n_2, r_2)c(n_1, r_1)\ c(n_0, r_0)^2}{c(n_0, r_0)c(n_1, r_1)}.$$

Thus $A_{2k}(n_2, r_2, 1)$ (under the same normalization as above) interpolates to a ratio of smooth functions (according to the already proven cases of the theorem!) on U. Passing, if necessary, to a smaller neighborhood $U_1 \subset U$ (in order to avoid the possible zeros in the denominator), we get the assertion of the theorem in this case as well. We remark that this was the only step in the whole proof, where the (technical) condition (5) was used.

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