On a Conjecture of Kohnen

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0. Introduction

In this paper we prove, under a certain additional condition, the conjecture proposed by Kohnen in [**8**]. This condition (see (5) below) does not seem to be too restrictive; the author does not know an example when it is not satisfied and it is easy to verify the condition in any specific case. The conjecture was based on the numerical evidence and conjectures from [**7**], and the theoretical result presented in [**8**]. A special case of a closely related problem was considered in [**11**].

To attack the conjecture we use the original results from [**8**] and the techniques for p-adic interpolation of cycle integrals created by Stevens [**12**].

We recall the setting of [**8**], formulate the conjecture and our results in the first section. In section 2, we recall certain notions from [**12**], quote the result from loc. cit. which we need, and specialize this to our setting. Finally, we prove our result in section 3.

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1. Statement of the result. In this section we briefly recall the setting and results from [**8**], and formulate our results. For the reader's conveniens we conserve all the notations from loc. cit. and refer to this article for the detailed discussion and explanations.

For integers $k \geq 1$ and $N \geq 1$ we denote by $J_{k+1,N}^{cusp}$ the space of Jacobi cusp forms of weight $k+1$ and index N [**2**]. Recall that a function $\phi \in J_{k+1,N}^{cusp}$ has a Fourier expansion of the form

$$\phi(\tau, z) = \sum_{n,r\in\mathbb{Z};\ r^2<4Nn} c(n,r) q^n \zeta^r \qquad (\tau \in \mathfrak{H} = \text{complex upper half-plane}, z \in \mathbb{C}, q = e^{2\pi i\tau}, \zeta = e^{2\pi i z}),$$

where $c(n,r)$ depends only on $r^2 - 4Nn$ and on the residue class r modulo $2N$.

Let $S_{2k}(N)^-$ denote the space of cusp forms of weight $2k$ on $\Gamma_0(N)$ with sign -1 in the functional equation of their L-series. For every normalized Hecke eigenform $f = \sum a(n)q^n \in S_{2k}^{new}(N)^-$ there exist ([**10**]) a non-zero Jacobi form $\phi \in J_{k+1,N}^{cusp,new}$ having the same Hecke eigenvalues and uniquely determined up to a non-zero scalar multiply. The Fourier expansions of f and ϕ are related by

$$c(n_0, r_0)a(n) = \sum_{d|n} \left(\frac{D_0}{d}\right) d^{k-1} c\left(\frac{n^2}{d^2}n_0, \frac{n}{d}r_0\right), \tag{1}$$

where $D_0 = r_0^2 - 4Nn_0 < 0$ is a fundamental discriminant and n is a positive integer.

We can and will assume that the Fourier coefficients $c(n,r)$ are algebraic numbers.

The Fourier coefficients of the Jacobi form ϕ can be expressed in the terms of cycle integrals [**3**]:

$$c(n,r)c(n_0,r_0) = \left(\frac{i}{2N}\right)^{k-1} \frac{||\phi||^2}{||f||^2} r_{k,N,DD_0,rr_0,D_0}(f), \tag{2}$$

where $D_0 = r_0^2 - 4Nn_0 < 0$ is a fundamental discriminant, and $||\phi||$ and $||f||$ denote the Petersson norm of $||\phi||$ and $||f||$, respectively. See Section 3, below, for the precise definition of the quantities $r_{k,N,DD_0,rr_0,D_0}(f)$.

Let $p \geq 5$ be a prime. Let K be an imaginary quadratic field with ring of integers O. In the following we fix an embedding $K \hookrightarrow \mathbb{C}$. We assume that our fixed prime p splits in K and that the class number of K is one, so that $(p) = \mathfrak{p}\overline{\mathfrak{p}} = (\rho)(\overline{\rho})$. We fix an embedding $\overline{\mathbb{Q}} \hookrightarrow \overline{\mathbb{Q}}_p$. The restricted map $K \hookrightarrow \mathbb{Q}_p$ induces the place $\mathfrak{p} = (\rho)$, thus $\overline{\rho}$ is a p-adic unit. Let E be an elliptic curve defined over $\mathbb{Q}$, with complex multiplication by O and let ψ be the Hecke character such that the Hasse-Weil L-function of E equals the L-function associated to ψ.

The function

$$F_k(\tau) = \sum_{\mathfrak{a}} \psi^{(2k-1)}(\mathfrak{a}) e^{2\pi i \mathcal{N}(\mathfrak{a})\tau}$$

is a cusp Hecke eigenform of weight $2k$. Here $\psi^{(2k-1)}$ represents the primitive Hecke character attached to ψ^{2k-1}.

From here on we fix a positive integer k_0 such that $F_{k_0} \in S_{2k_0}^{new}(N)^-$. For any positive $k \equiv k_0 \bmod p-1$, we have $F_k \in S_{2k}^{new}(N)^-$.

Following [**8**], for fundamental discriminants $D_1 = r_1^2 - 4Nn_1 < 0$ and $D_2 = r_2^2 - 4Nn_2 < 0$, and a positive integer k, define

$$Z_{n_1,r_1,n_2,r_2;k_0}(k) = \left(\frac{2\pi}{\Omega_\infty}\right)^{2k-1} \left(\frac{i}{\sqrt{d}}\right)^{k-1} r_{k,N,D_1D_2,r_1r_2,D_1}(F_k), \tag{3}$$

and

$$Z^*_{n_1,r_1,n_2,r_2;k_0}(k) = \left(1 - \left(\frac{D_1}{p}\right)\rho^{2k-1}p^{-k}\right)\left(1 - \left(\frac{D_2}{p}\right)\rho^{2k-1}p^{-k}\right) Z_{n_1,r_1,n_2,r_2;k_0}(k), \tag{4}$$

where Ω_∞ is the complex period of E chosen as in [**8**]. It is proven in [**8**, Proposition 1], that $Z_{n_1,r_1,n_2,r_2;k_0}(k)$ is rational and p-integral if $(D_1D_2, Np) = 1$; these numbers are the main subject of our consideration.

Since E has good ordinary reduction at p one can define the p-adic period ω_p. The construction uses the isomorphism between the formal group $\hat{E}$ of the elliptic curve $E/\mathbb{Q}_p$ and the formal multiplicative group $\mathbb{G}_m$. We refer to [**8**],[**1**] for the precise definition.

Our result is closely related to the following conjecture of Kohnen [**8**].

Conjecture. *Suppose that $(D_1D_2, Np) = 1$. Let k and k' be natural numbers congruent to k_0 modulo $p-1$. Then for all $n \geq 1$ the congruence*

$$\omega_p^{2k-1} Z^*_{n_1,r_1,n_2,r_2;k_0}(k) \equiv \omega_p^{2k'-1} Z^*_{n_1,r_1,n_2,r_2;k_0}(k') \bmod p^n$$

holds, if $k \equiv k' \bmod p^{n-1}$.

Let $\phi_k = \sum c_k(n,r) q^n \zeta^r$ be the Jacobi form associated with F_k.

We are now ready to formulate our main result.

Theorem 1 *Suppose that $(D_1D_2, Np) = 1$. Let k and k' be natural numbers congruent to k_0 modulo $p-1$. Assume that there exist $D_0 \equiv 0 \bmod p$ with $(D_0, N) = 1$, and $k_1 \equiv k_0 \bmod p-1$ such that*

$$c_{k_1}(n_0, r_0) \neq 0 \tag{5}$$

Then for all $n \geq 1$ the congruence

$$\omega_p^{2k-1} Z^*_{n_1,r_1,n_2,r_2;k_0}(k) \equiv \omega_p^{2k'-1} Z^*_{n_1,r_1,n_2,r_2;k_0}(k') \bmod p^n$$

holds, if $k \equiv k' \bmod p^{n-1}$.

Remarks.

1. In the case $\left(\frac{D_1D_2}{p}\right) = 1$, the conjecture (and hence our Theorem 1) was proven by Kohnen in the original paper [**8**] without additional assumptions. This was the main result of loc. cit., and it supported the conjecture.
2. Making use of (1) and the fact that F_k is a Hecke eigenform, one easily derives from Theorem 1 the corresponding congruences for arbitrary (i.e. not fundamental) discriminants D_1 and D_2 still coprime to Np (cf. [**8**, Remark ii, p. 274]).
3. In fact, we prove that the quantity $\omega_p^{2k-1} Z^*_{n_1,r_1,n_2,r_2;k_0}(k)$ interpolates to an Iwasawa function, which yields Theorem 1.

2. p-adic continuation of the cycle integrals. In this section, we formulate certain results from [**12**] in the specialized form which is convenient for our purposes.

Let $\Lambda = \mathbb{Z}_p[[1 + p\mathbb{Z}_p]]$ be the completed group ring on the principal unit group $1 + p\mathbb{Z}_p$, and $\Lambda_N = \mathbb{Z}_p[[\mathbb{Z}_{p,N}]]$, where $\mathbb{Z}_{p,N} = \lim_{\leftarrow} (\mathbb{Z}/Np^m\mathbb{Z})^*$. One has the natural isomorphism $\Lambda_N \simeq \Lambda[(\mathbb{Z}/pN\mathbb{Z})^*]$. Following [**12**], consider the Λ_N - (and, therefore, Λ -) algebra $\mathcal{R}_N$, which is the universal p-ordinary Hecke algebra of tame conductor N. The universal p-ordinary metaplectic Hecke algebra of tame conductor N is defined by (loc. cit.)

$$\tilde{\mathcal{R}}_N = \mathcal{R}_N \otimes_{\Lambda_N,\sigma} \Lambda_N.$$

Here $\sigma : \Lambda_N \to \Lambda_N$ is the map induced by the continuous group homomorphism $t \mapsto t^2$ on $\mathbb{Z}_{p,N} \to \mathbb{Z}_{p,N}$. For any $k \geq 1$ with $k \equiv k_0 \bmod p-1$, the modular form

$$F_k^*(\tau) = F_k(\tau) - \rho^{2k-2} F_k(p\tau)$$

is a p-stabilized ordinary newform of tame conductor N (cf. [**12**], [**4**]). Moreover, it follows from [**5**, Section 7.2], that there is a Λ-adic p-ordinary Hecke eigenform $F(X)$ such that $F(u^{2k-2}-1)=F_k^*$. Here and in the following we chose the topological generator $u=1+p$ of $(1+p\mathbb{Z}_p)^*$.

Thus, there is a direct sum decomposition corresponding to $F(X)$:

$$\mathcal{R}_N \simeq \Lambda_N \oplus \mathcal{R}'_N, \tag{6}$$

and it follows that

$$\tilde{\mathcal{R}}_N \simeq (\Lambda_N \otimes_{\Lambda_N,\sigma} \Lambda_N) \oplus \tilde{\mathcal{R}}'_N. \tag{7}$$

We consider an element $J \in \tilde{\mathcal{R}}_N$ as an element of Λ_N via the projection of (7) to the first component, and the map

$$\Lambda_N \otimes_{\Lambda_N,\sigma} \Lambda_N \to \Lambda_N,$$

which takes $\alpha \otimes 1$ to α^2.

We will identify a positive integer k with the arithmetic point in $\mathcal{X}(\Lambda_N) = Hom_{cont}(\Lambda_N, \overline{\mathbb{Q}}_p)$ which sends $t \in \mathbb{Z}_{p,N}^*$ into t^k.

For an element $J \in \Lambda_N$ we write $J(k)$ for the value of J at k (considered as an element of $\mathcal{X}(\Lambda_N)$. The elements of Λ_N are p-adic analytic functions on $\mathcal{X}(\Lambda_N)$.

If $J \in \mathcal{X}(\Lambda_N)$, and for any $y \in \Lambda_N$ we have $J(y) \in \mathcal{O}_p = \{x \in \overline{\mathbb{Q}}_p,\ |x|_p \le 1\}$, then the Kummer congruences hold. Namely, $k \equiv k' \bmod (p-1)p^n$ with $n \ge 0$ yields $|J(k)-J(k')|_p \le p^{-n-1}$.

For a cusp form f of weight $2k$, the cycle integral is defined by

$$r_{k,N,Q}(f) = \int_{C_Q} f(\tau)Q(\tau,1)^{k-1}d\tau,$$

where C_Q is the image in $\Gamma_0(N)\backslash\mathfrak{H}$ of the semi-circle $a|\tau|^2 + b\Re(\tau) + c = 0$ oriented from $(-b-\sqrt{\Delta})/2a$ to $(-b+\sqrt{\Delta})/2a$, if $a \ne 0$, or the vertical line $bx+c=0$, oriented from $-c/b$ to $i\infty$ if $b>0$, and from $i\infty$ to $-c/b$ if $b<0$, if $a=0$.

The proposition below is a reformulation of a special case of [**12**, Theorem 5.5] and [**12**, Lemma 6.1]. For a quadratic form $Q=[a,b,c]=ax^2+bxy+cy^2$ we put $Q^\iota=[a,-b,c]=ax^2-bxy+cy^2$.

Proposition 1. *There exist*
- *complex numbers* $\Omega^-(k) \in \mathbb{C}^*$
- *p-adic periods* $\Omega_k \in \overline{\mathbb{Q}}_p$ *with* $\Omega_{k_0} \ne 0$
- *an element* $J_Q \in \Lambda_N$ *defined for any integral quadratic form* $Q=[a,b,c]$ *with* $a \equiv b \equiv 0 \bmod p$ *such that*

$$J_Q(k) = \frac{\Omega_k}{\Omega^-(k)}(r_{k,N,Q}(F_k^*) + r_{k,N,Q^\iota}(F_k^*)).$$

3. Proof of Theorem 1. We keep all notations and conventions of the previous sections. In particular, we fix the fundamental discriminant $D_0 = r_0^2 - 4Nn_0 < 0$ such that $D_0 \equiv 0 \bmod p$ and $c_{k_0}(n_0,r_0) \ne 0$.

The quantity $r_{k,N,DD_0,rr_0,D_0}(f)$, appearing in (2), is defined as a weighted sum of cycle integrals:

$$r_{k,N,DD_0,rr_0,D_0}(f) = \sum_{Q \in \mathcal{Q}_{N,DD_0,rr_0}/\Gamma_0(N)} \chi_{D_0}(Q) r_{k,N,Q}(f). \tag{8}$$

Let R be a residue modulo $2N$ and $\Delta > 0$ a discriminant with $\Delta \equiv R^2 \bmod 4N$. Consider the set of integral binary quadratic forms

$$\mathcal{Q}_{N,\Delta,R} = \left\{Q(x,y) = ax^2+bxy+cy^2 |\ a,b,c \in \mathbb{Z}, b^2-4ac = \Delta, a \equiv 0(N), b \equiv R(2N)\right\}.$$

The group $\Gamma_0(N)$ acts on $\mathcal{Q}_{N,\Delta,R}$ by $\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \circ Q(x,y) = Q(\alpha x+\beta y, \gamma x+\delta y)$. The number of equivalence classes is finite. For a fundamental discriminant D_0 dividing Δ we denote by $\chi_{D_0} : \mathcal{Q}_{N,\Delta,R}/\Gamma_0(N) \to \pm 1, 0$ the generalized genus character ([**3**, Chapter I, Section 2]).

Let $D_1 = r_1^2 - 4Nn_1 \not\equiv 0 \bmod p$ be a negative fundamental discriminant. We claim that there exist $r(D_0, D_1) \in \Lambda_N$ such that

$$r(D_0,D_1)(k) = \frac{\Omega_k}{\Omega^-(k)}\left(1 - p^{-k}\rho^{2k-1}\left(\frac{D_1}{p}\right)\right) r_{k,N,D_0D_1,r_0r_1,D_0}(F_k). \tag{9}$$

To prove the claim, we are going to assemble the right hand side out of the elements $J_Q \in \Lambda_N$. We need some preparations connected with the systems of representatives for $\mathcal{Q}_{N,\Delta,R}/\Gamma_0(N)$

Lemma 1.
a. *Assume $\Delta \equiv 0 \bmod p$. Then in any class of $\mathcal{Q}_{N,\Delta,R}/\Gamma_0(N)$ one can choose a representative $Q = [a, b, c]$ such that $a \equiv b \equiv 0 \bmod p$.*
b. *Assume in addition that $\Delta \not\equiv 0 \bmod p^2$. If $Q_i = [a_i, b_i, c_i]$ is a system of representatives for $\mathcal{Q}_{N,\Delta,R}/\Gamma_0(N)$, satisfying $a_i \equiv b_i \equiv 0 \bmod p$, then $P_i = [a_i/p, b_i, c_ip]$ is also a system of representatives.*
c. *Put $\Delta = D_0D_1$. Then for any $[a, b, c] \in \mathcal{Q}_{N,\Delta,R}$, with $a \equiv b \equiv 0 \bmod p$,*

$$\chi_{D_0}([a,b,c]) = \left(\frac{D_1}{p}\right)\chi_{D_0}([a/p,b,cp])$$

One proves **a** and **b** by a straightforward argument, which we omit. Part **c** follows from the explicit formula for the genus character ([**3**, Proposition 1, P3]).

If $a \neq 0$ put

$$M_Q = \begin{pmatrix} (v-bu)/2 & -cu \\ au & (v+bu)/2 \end{pmatrix},$$

where (v, u) is the smallest positive solution of the Pell's equation $x^2 - \Delta y^2 = 4$. Then

$$r_{k,N,Q}(f) = \int_{i\infty}^{M_Q i\infty} f(\tau)Q(\tau,1)^{k-1}d\tau.$$

The definition (8) does not depend on the specific choice of the system of representatives $Q \in \mathcal{Q}_{N,D_0D_1,rr_0}$. We pick such a system as in **a** of Lemma 1. It is sufficient to prove

$$r(D_0,D_1)(k) = \frac{\Omega_k}{\Omega^-(k)}\sum_{Q\in\mathcal{Q}_{N,DD_0,rr_0}/\Gamma_0(N)}\chi_{D_0}(Q)r_{k,N,Q}(F_k^*), \tag{10}$$

because Proposition 1 imlpies that the sum in the right is an evaluation at k of an element from Λ_N. (Note that $\chi_{D_0}(Q) = \chi_{D_0}(Q^\iota)$. Moreover, when Q runs through a system of representatives from $\mathcal{Q}_{N,DD_0,rr_0}/\Gamma_0(N)$ the same happens with Q^ι.) Indeed,

$$\frac{\Omega_k}{\Omega^-(k)}\sum_{Q\in\mathcal{Q}_{N,D_0D_1,r_0r_1}/\Gamma_0(N)}\chi_{D_0}(Q)r_{k,N,Q}(F_k^*)$$

$$= \frac{\Omega_k}{\Omega^-(k)}\sum_{Q\in\mathcal{Q}_{N,D_0D_1,r_0r_1}/\Gamma_0(N)}\chi_{D_0}\int_{i\infty}^{M_Q i\infty}(F_k(\tau) - \rho^{2k-1}F_k(p\tau))Q(\tau,1)^{k-1}d\tau$$

$$= \frac{\Omega_k}{\Omega^-(k)}r_{k,N,D_0D_1,r_0r_1,D_0}(F_k) - \rho^{2k-1}\sum_{Q\in\mathcal{Q}_{N,D_0D_1,r_0r_1}/\Gamma_0(N)}\chi_{D_0}(Q)\int_{i\infty}^{M_Q i\infty}F_k(p\tau)Q(\tau,1)^{k-1}d\tau$$

Part **b** of the Lemma 1 shows that after the variable change $\tau \mapsto \tau/p$ the sum is still over a system of representatives, and part **c** controls the quantities $\chi_{D_0}(Q)$. We have after that

$$\sum_{Q\in\mathcal{Q}_{N,D_0D_1,r_0r_1}/\Gamma_0(N)}\chi_{D_0}(Q)\int_{i\infty}^{M_Q i\infty}F_k(p\tau)Q(\tau,1)^{k-1}d\tau$$

$$= p^{-k}\left(\frac{D_1}{p}\right)\frac{\Omega_k}{\Omega^-(k)}r_{k,N,D_0D_1,r_0r_1,D_0}(F_k).$$

This proves (10) and, therefore, our claim that $r(D_0, D_1) \in \Lambda_N$.

It might happen that $r(D_0, D_1) = 0$. It follows from (2), (9), (3), (4) and our assumption (5) that in this case the quantity $Z^*_{n_1,r_1,n_2,r_2;k_0}(k)$, considered in Theorem 1, vanishes for any n_2, r_2, and k, and we have nothing to prove in this case. Thus we assume $r(D_0, D_1)$ to be non-zero.

By the proof of [**8**, Proposition 1, p. 273], we have

$$Z_{n_1,r_1,n_1,r_1;k_0}(k) = \left(\frac{2\pi}{\sqrt{d}}\right)^{k-1} \frac{(k-1)!}{(\omega_\infty/\sqrt{|D_1|})^{2k-1}} L(\psi^{2k-1}, D_1, k), \tag{11}$$

where $-d$ is the discriminant of K, and $L(\psi^{2k-1}, D, s)$ is $L(\psi^{2k-1}, s)$ twisted with the character $n \mapsto \left(\frac{D}{\mathcal{N}(n)}\right)$, as in [**8**, p.272]. The existence of the bounded p-adic measure constructed by Katz [**8**, p.272, (7)] yields the existence of $Z_{1,1} \in \Lambda$ such that

$$Z_{1,1}(k) = \omega_p^{2k-1} Z^*_{n_1,r_1,n_1,r_1;k_0}(k) \tag{12}$$

as soon as $k \equiv k_0 \bmod p-1$.

Pick a fundamental discriminant $D_2 = r_2^2 - 4Nn_2 < 0$ with $(D_2, Np) = 1$.

We apply the same argument also to

$$Z_{n_1,r_1,n_2,r_2;k_0}(k)^2 = \left(\frac{2\pi}{\sqrt{d}}\right)^{2k-2} \frac{(k-1)!}{(\omega_\infty/\sqrt{|D_1|})^{2k-1}} \frac{(k-1)!}{(\omega_\infty/\sqrt{|D_2|})^{2k-1}} L(\psi^{2k-1}, D_1, k) L(\psi^{2k-1}, D_2, k)$$

instead of (11). This implies that there exist $Z^{(2)}_{1,2} \in \Lambda$ such that

$$Z^{(2)}_{1,2}(k) = \omega_p^{4k-2} Z^*_{n_1,r_1,n_2,r_2;k_0}(k)^2 \tag{13}$$

We remark that in order to prove, for instance, the existence of $Z_{1,1} \in \Lambda$, we cannot refer directly to [**8**, Corollary 1] since the analytic function constructed in loc. cit. might be not an Iwasawa function.

One has by (2), (9), (3), (4) and (12)

$$Z_{1,1}(k) \frac{r(D_0, D_2)(k)}{r(D_0, D_1)(k)} = \omega_p^{2k-1} Z^*_{n_1,r_1,n_2,r_2;k_0}(k). \tag{14}$$

By the p-adic Weierstraß preparation theorem [**13**, Theorem 7.3] one can rewrite the left-hand side as

$$p^\mu U(T) \frac{A(T)}{B(T)} = Z_{1,1} \frac{r(D_0, D_2)}{r(D_0, D_1)}, \tag{15}$$

where $\mu \in \mathbb{Z}$, the series $U(T) \in \mathbb{Z}[[T]]$ is a unit, and $A, B \in \mathbb{Z}[T]$ are distinguished polynomials. Here we identify the rings $\mathbb{Z}_p[[1+p\mathbb{Z}_p]] \simeq \mathbb{Z}_p[[T]]$ sending $u = 1+p$ to T (cf. [**13**, Theorem 7.1]). Making use of this observation, taking the square of (14), and taking (13) into account, we get

$$p^{2\mu} U(T)^2 \frac{A(T)^2}{B(T)^2} = Z^{(2)}_{1,2}(T).$$

It now follows that $p^{2\mu} A(T)^2 / B(T)^2 \in \mathbb{Z}_p[[T]]$. Since the square of a distinguished polynomial is a distinguished polynomial, we get by [**13**, Lemma 7.5] that $B(T)^2$ divides $p^{2\mu} A(T)^2$ as polynomials, and, therefore, $B(T)$ also divides $p^\mu A(T)$, i.e. $p^\mu A(T)/B(T) \in \mathbb{Z}_p[T]$. Combining this observation with (15), we find that the left-hand side of (14) belongs to Λ. In other words, there exists a power series $R(T) \in \mathbb{Z}_p[[T]]$ such that

$$R(u^k - 1) = Z_{1,1}(k) \frac{r(D_0, D_2)(k)}{r(D_0, D_1)(k)} = \omega_p^{2k-1} Z^*_{n_1,r_1,n_2,r_2;k_0}(k)$$

if $k \equiv k_0 \bmod p-1$. The Kummer congruences claimed in Theorem 1 follow from this.

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