

PRACTICE PROBLEMS 2

- (1) A cylindrical can without a top is made to contain  $20\text{cm}^3$  of liquid. The metal for the circular base costs  $\$2/\text{cm}^2$  and the metal for the sides costs  $\$1/\text{cm}^2$ . Find the dimensions that will minimize the cost of the metal to make the can, and also the value of the minimum cost.
- (2) Show that of all the rectangles with a given area  $A$ , the one with the smallest perimeter is a square.
- (3) A plane is flying horizontally at an altitude of 1mi. and a ground speed of 500mi/hr passes directly over a radar station. Find the rate at which the distance to the radar station is increasing when it is 2mi. away from the station.
- (4) Find  $y'$  using implicit differentiation:  
 $y^3 + y^2 - x^3 = 5$
- (5) Find the following indefinite integrals:

(i)

$$\int \sqrt{1+x^2} x^5 dx$$

(ii)

$$\int \frac{e^x}{e^x + 1} dx$$

(iii)

$$\int \frac{e^{\sqrt{x}+1}}{\sqrt{x}} dx$$

- (6) Evaluate the definite integrals:

(i)

$$\int_1^2 x\sqrt{x-1} dx$$

(ii)

$$\int_0^4 \frac{x}{\sqrt{1+2x}} + x dx$$

(iii)

$$\int_0^2 \frac{dx}{(2x-3)^2} dx$$

- (7) For the following functions find:

- (a) the intervals of increase and decrease  
(b) the relative(local) extrema  
(c) the intervals of concavity  
(d) inflection points

(i)  $f(x) = 1 - 3x + 5x^2 - x^3$

(ii)  $f(x) = x^4 - 6x^2$

(iii)  $f(x) = \ln(1 + x^2)$

- (8) Find the absolute maximum and absolute minimum of the following functions on the given intervals:

(i)  $f(x) = \sqrt{9 - x^2}$  on  $[-1, 2]$

(ii)  $f(x) = x^2 + \frac{2}{x}$  on  $[\frac{1}{2}, 2]$