

PRACTICE PROBLEMS 2

- (1) A cylindrical can without a top is made to contain 20cm^3 of liquid. The metal for the circular base costs $\$2/\text{cm}^2$ and the metal for the sides costs $\$1/\text{cm}^2$. Find the dimensions that will minimize the cost of the metal to make the can, and also the value of the minimum cost.

$$\boxed{r = \left(\frac{10}{\pi}\right)^{\frac{1}{3}}}, \quad \boxed{h = \frac{20\pi^{\frac{2}{3}}}{\pi 10^{\frac{2}{3}}} = \frac{20}{\pi^{\frac{1}{3}} 10^{\frac{2}{3}}}}, \quad \boxed{\text{Cost} \approx \$41}$$

- (2) Show that of all the rectangles with a given area A , the one with the smallest perimeter is a square.
- (3) A plane is flying horizontally at an altitude of 1mi. and a ground speed of 500mi/hr passes directly over a radar station. Find the rate at which the distance to the radar station is increasing when it is 2mi. away from the station.
- (4) Find y' using implicit differentiation:
 $y^3 + y^2 - x^3 = 5$
- (5) Find the following indefinite integrals:

(i)

$$\int \sqrt{1+x^2} x^5 dx$$

Let $u = 1 + x^2$ then $du = 2x dx$, $\frac{du}{2} = x dx$, $x^2 = u - 1$, and we get

$$= \int \sqrt{u} x^2 x dx = \int u^{1/2} (u-1)(u-1) \frac{du}{2}$$

Multiply it out, integrate, and plug back in for u and get:

$$\boxed{\frac{1}{2} \left(\frac{2}{7} (1+x^2)^{7/2} - \frac{4}{5} (1+x^2)^{5/2} + \frac{2}{3} (1+x^2)^{3/2} \right) + C}$$

(ii)

$$\int \frac{e^x}{e^x + 1} dx$$

Let $u = e^x + 1$, then $du = e^x dx$, (also $x = \ln(u-1)$, $dx = \frac{1}{u-1} du$)

$$= \int \frac{1}{e^x + 1} (e^x dx) = \int \frac{1}{u} du$$

then we get $\boxed{\ln(e^x + 1) + C}$

(iii)

$$\int \frac{e^{\sqrt{x}+1}}{\sqrt{x}} dx$$

Let $u = \sqrt{x} + 1$, $= \boxed{2e^{\sqrt{x}+1} + C}$.

- (6) Evaluate the definite integrals:

(i)

$$\int_1^2 x\sqrt{x-1} dx$$

Let $u = x - 1$, then $x = 1 + u$, $dx = du$:

$$\int (u+1)\sqrt{u} du = \int u^{\frac{3}{2}} + u^{\frac{1}{2}} du = \frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C = \frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C$$

$$\int_1^2 x\sqrt{x-1} = \frac{2}{5}((2)-1)^{5/2} + \frac{2}{3}((2)-1)^{3/2} - \left[\frac{2}{5}((1)-1)^{5/2} + \frac{2}{3}((1)-1)^{3/2} \right] = \boxed{\frac{16}{15}}$$

(ii)

$$\begin{aligned} & \int_0^4 \frac{x}{\sqrt{1+2x}} + x \, dx \\ &= \int_0^4 \frac{x}{\sqrt{1+2x}} \, dx + \int_0^4 x \, dx \end{aligned}$$

Then $\int_0^4 x \, dx = \frac{1}{2}x^2 \Big|_0^4 = 8$, and we can calculate the other integral by letting $u = 1 + 2x$, then $x = \frac{u-1}{2}$, $dx = \frac{1}{2}du$,

$$\int \frac{x}{\sqrt{1+2x}} \, dx = \int \frac{\frac{u-1}{2}}{u^{1/2}} \frac{du}{2} = \frac{1}{4} \int u^{3/2} - u^{1/2} \, du = \frac{1}{4} \left[\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2} \right] + C = \frac{1}{10}(1+2x)^{5/2} + \frac{1}{6}(1+2x)^{3/2} + C$$

$$\int_0^4 \frac{x}{\sqrt{1+2x}} + x \, dx = \left[\frac{1}{10}(9)^{5/2} + \frac{1}{6}(9)^{3/2} \right] - \left[\frac{1}{10}(1)^{5/2} + \frac{1}{6}(1)^{3/2} \right] + 8$$

(iii)

$$\int_0^2 \frac{dx}{(2x-3)^2}$$

(7) For the following functions find:

- (a) the intervals of increase and decrease
- (b) the relative(local) extrema
- (c) the intervals of concavity
- (d) inflection points
- (i) $f(x) = 1 - 3x + 5x^2 - x^3$
- (ii) $f(x) = x^4 - 6x^2$
- (iii) $f(x) = \ln(1 + x^2)$

(8) Find the absolute maximum and absolute minimum of the following functions on the given intervals:

- (i) $f(x) = \sqrt{9 - x^2}$ on $[-1, 2]$
- (ii) $f(x) = x^2 + \frac{2}{x}$ on $[\frac{1}{2}, 2]$