Math 215 - Derivatives Answers/Solutions

1. $f'(x) = \frac{1}{3}x^{-\frac{2}{3}} + (-\frac{1}{3})x^{-\frac{4}{3}} = \frac{1}{3}x^{-\frac{2}{3}}(1 - x^{-\frac{2}{3}})$
2. $f'(x) = 8(x+2)^7(x+3)^6 + (x+2)^8 6(x+3)^5 = 2(x+2)^7(x+3)^5(7x+18)$
3. $f'(x) = \dfrac{\sqrt{9-4x} - 2x(\frac{1}{2})(9-4x)^{-\frac{1}{2}}(-4)}{(\sqrt{9-4x})^2} = \dfrac{(9-4x)^{\frac{1}{2}} + 2x(9-4x)^{-\frac{1}{2}}}{9-4x}$
4. $f'(x) = \sqrt{7}\left(x + \dfrac{1}{x^2}\right)^{\sqrt{7}-1}(1 - 2x^{-3})$
5. $f'(x) = -1(1-x^{-1})^{-2}(-(-1)x^{-2}) = -(1-x^{-1})^{-2}(x^{-2})$
6. $f'(\theta) = 2\sec(2\theta)\tan(2\theta)$
7. $f'(t) = -2(-\frac{4}{3}t^{-\frac{7}{3}}) = \frac{8}{3}t^{-\frac{7}{3}}$
8. $f'(x) = \frac{1}{5}(\tan x)^{-\frac{4}{5}}(\sec^2 x)$
9. $f'(x) = \cos(\cos x)(-\sin x) = -\cos(\cos x)(\sin x)$
10. $f'(x) = -\dfrac{1}{3}(x + x^{\frac{1}{2}})^{-\frac{4}{3}}(1 + \dfrac{1}{2}x^{-\frac{1}{2}})$
11. $f(x) = \dfrac{(x-1)(x-4)}{(x-2)(x-3)} = \dfrac{x^2-5x+4}{x^2-5x+6}$

$$\begin{aligned} f'(x) &= \frac{(x^2+5x+6)(2x-5) - (x^2-5x+4)(2x-5)}{(x^2-5x+6)^2} \\ &= \frac{(2x-5)(x^2+5x+6-x_5^2x-4)}{(x^2-5x+6)^2} \\ &= \frac{(2x-5)(10x+2)}{(x^2-5x+6)^2} \end{aligned}$$

12. $f'(x) = \frac{1}{2}(\sin\sqrt{x})^{-\frac{1}{2}}(\frac{1}{2}(x^{-\frac{1}{2}})) = \frac{1}{4}x^{-\frac{1}{2}}(\sin\sqrt{x})^{\frac{1}{2}} = \frac{1}{4\sqrt{x}}\sqrt{\sin\sqrt{x}}$
13. $f'(x) = (\sec^2\sqrt{1-x})(\frac{1}{2}(1-x)^{-\frac{1}{2}})(-1) = -\frac{1}{2}\sqrt{1-x}(\sec^2\sqrt{1-x})$
14. $f'(x) = -1(\sin(x-\sin x))^{-2}(\cos(x-\sin x))(1-\cos x)$
15. $f'(x) = \cos(\tan\sqrt{1+x^3})(\sec^2\sqrt{1+x^3})(\frac{1}{2}(1+x^3)^{-\frac{1}{2}})(3x^2) =$
$\frac{3x^2}{2\sqrt{1+x^3}}(\sec^2\sqrt{1+x^3})(\cos(\tan\sqrt{1+x^3}))$

16.

$$\begin{aligned} f'(x) &= \frac{(x^4+\lambda^4)4(x+\lambda)^3-(x+\lambda)^4(4x^3)}{(x^4+\lambda^4)^2} \\ &= \frac{4(x+\lambda)^3(x^4+\lambda^4-(x+\lambda)(x^3))}{(x^4+\lambda^4)^2} \\ &= \frac{4(x+\lambda)^3(\lambda^4-\lambda x^3)}{(x^4+\lambda^4)^2} \end{aligned}$$

17. $f'(x) = -\csc^2(3x^2+5)(6x) = -6x\csc(3x^2+5)$

18. $f'(x) = \dfrac{mx\cos mx - \sin mx \cdot 1}{x^2} = \dfrac{mx\cos(mx)-\sin(mx)}{x^2}$

19. $f'(x) = 2\cos(\tan x)(-\sin(\tan x))(\sec^2 x) = -2\cos(\tan x)(\sin(\tan x))(\sec^2 x)$

20.

$$\begin{aligned} \frac{d}{dx}(x^2y^3+3y^2 &= x-4y) \\ 2xy^3+x^2 3y^2\frac{dy}{dx}+6y\frac{dy}{dx} &= 1-4\frac{dy}{dx} \\ x^2 3y^2\frac{dy}{dx}+6y\frac{dy}{dx}+4\frac{dy}{dx} &= 1-2xy^3 \\ \frac{dy}{dx}(3x^2y^2+6y+4) &= 1-2xy^3 \\ \frac{dy}{dx} &= \frac{1-2xy^3}{3x^2y^2+6y+4} \end{aligned}$$

21.

$$\begin{aligned} \frac{d}{dx}(y\sqrt{x-1}+x\sqrt{y-1} &= xy) \\ \frac{dy}{dx}\sqrt{x-1}+y(\frac{1}{2}(x-1)^{-\frac{1}{2}}+1\cdot\sqrt{y-1}+x(\frac{1}{2}(y-1)^{-\frac{1}{2}})\frac{dy}{dx} &= x\frac{dy}{dx}+y \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx}\sqrt{x-1}+\frac{x}{2}(y-1)^{-\frac{1}{2}}\frac{dy}{dx}-x\frac{dy}{dx} &= y-\frac{y}{2}(x-1)^{-\frac{1}{2}}-\sqrt{y-1} \\ \frac{dy}{dx}(\sqrt{x-1}+\frac{x}{2}(y-1)^{-\frac{1}{2}}-x) &= y-\frac{y}{2}(x-1)^{-\frac{1}{2}}-\sqrt{y-1} \\ \frac{dy}{dx} &= \frac{y-\frac{y}{2}(x-1)^{-\frac{1}{2}}-\sqrt{y-1}}{\sqrt{x-1}+\frac{x}{2}(y-1)^{-\frac{1}{2}}-x} \end{aligned}$$

22.

$$\begin{aligned} \frac{d}{dx}(x^2 = y(y+1) &= y^2+y) \\ 2x &= 2y\frac{dy}{dx}+\frac{dy}{dx} \\ \frac{dy}{dx} &= \frac{2x}{2y+1} \end{aligned}$$

23.

$$\begin{aligned}
\frac{d}{dx}(x\tan y &= y-1)\\
x\sec^2 y\frac{dy}{dx}+\tan y &= \frac{dy}{dx}\\
\frac{dy}{dx}(1-x\sec^2 y) &= \tan y\\
\frac{dy}{dx} &= \frac{\tan y}{1-xsec^2y}
\end{aligned}$$