

# Math 215 - Integrals

## General Rules

1. Easy antiderivatives:  $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ ,  $\int e^x dx = e^x + C$ ,  $\int \frac{1}{x} dx = \ln|x| + C$ , etc.
2. Substitution: Let  $u = g(x)$  then  $du = g'(x)dx$ . Where  $g(x)$  is some function that can be found in the integral. Don't forget to check your bounds of integration. Make sure to change them to  $u$  or if you keep them as  $x$  to switch the  $u$ 's back to  $x$  after the integration is completed.
3. Integration By Parts:  $\int u dv = uv - \int v du$ . Let  $u = g(x)$  then differentiate to get  $du = g'(x)dx$ . Let  $dv = h(x)dx$  then integrate to get  $v = \int h(x)dx$ .

Remember the picture pattern:

Differentiate and integrate: 
$$\begin{array}{ccccc} & & u & dv & \\ & & \downarrow & \downarrow & \text{integrate} \\ \text{differentiate} & & du & v & \end{array}$$

Use the picture to complete the integral: 
$$\begin{array}{ccc} u & & dv \\ & \searrow & \\ du & \leftarrow & v \end{array}$$

then,  $\int u dv = uv - \int v du$

## Practice Integrals

1.  $\int_0^3 (5x^2 - 4x + 3) dx$

2.  $\int_1^3 \left(x + \frac{1}{x}\right)^2 dx$

3.  $\int \frac{x^4 - 1}{x^2 + 1} dx$

4.  $\int \frac{t^6 - t^4}{t^4} dt$

5.  $\int (x^3 - 1)^2 dx$

6.  $\int_2^4 (x - 1)(3x + 2) dx$

7.  $\int_0^1 (\sqrt[4]{x^5} + \sqrt[5]{x^4}) dx$

8.  $\int_0^1 x^{\frac{1}{2}}x^3 dx$
9.  $\int_0^{(\frac{\pi}{2})^2} \frac{\sin \sqrt{x}}{\sqrt{x}} dx$
10.  $\int_0^1 \frac{4}{(1+2x)^3} dx$
11.  $\int 2x(x^2+3)^4 dx$
12.  $\int x^3(1-x^4)^5 dx$
13.  $\int_1^2 \sqrt{x-1} dx$
14.  $\int x(x^2+1)^{\frac{3}{2}} dx$
15.  $\int \frac{1}{(1-3t)^4} dt$
16.  $\int \sqrt{3-5y} dy$
17.  $\int (\ln x)^2 dx$  Hint: use integration by parts two times.
18.  $\int t^3 e^t dt$  Hint: use integration by parts three times.
19.  $\int x e^{2x} dx$
20.  $\int x \cos x dx$
21.  $\int_0^1 x e^{-x} dx$
22.  $\int \frac{\ln x}{x^2} dx$
23.  $\int x \ln x dx$
24.  $\int x^2 \cos x dx$  Hint: use integration by parts two times.
25.  $\int \sqrt{x} \ln x dx$