

SERIES CONVERGENCE/DIVERGENCE FLOW CHART

TEST FOR DIVERGENCE

Does $\lim_{n \rightarrow \infty} a_n = 0$? NO $\rightarrow \sum a_n$ Diverges

YES

p-SERIES

Does $a_n = 1/n^p, n \geq 1$? YES \rightarrow Is $p > 1$? YES $\rightarrow \sum a_n$ Converges
NO $\rightarrow \sum a_n$ Diverges

NO

GEOMETRIC SERIES

Does $a_n = ar^{n-1}, n \geq 1$? YES \rightarrow Is $|r| < 1$? YES $\rightarrow \sum_{n=1}^{\infty} a_n = \frac{a}{1-r}$
NO $\rightarrow \sum a_n$ Diverges

NO

ALTERNATING SERIES

Does $a_n = (-1)^n b_n$ or $a_n = (-1)^{n-1} b_n, b_n \geq 0$? YES \rightarrow Is $b_{n+1} \leq b_n$ & $\lim_{n \rightarrow \infty} b_n = 0$? YES $\rightarrow \sum a_n$ Converges

NO

TELESCOPING SERIES

Do subsequent terms cancel out previous terms in the sum? May have to use partial fractions, properties of logarithms, etc. to put into appropriate form. YES \rightarrow Does $\lim_{n \rightarrow \infty} s_n = s$ s finite? YES $\rightarrow \sum a_n = s$
NO $\rightarrow \sum a_n$ Diverges

NO

TAYLOR SERIES

Does $a_n = \frac{f^{(n)}(a)}{n!} (x-a)^n$? YES \rightarrow Is x in interval of convergence? YES $\rightarrow \sum_{n=0}^{\infty} a_n = f(x)$
NO $\rightarrow \sum a_n$ Diverges

NO

Try one or more of the following tests:

COMPARISON TEST

Pick $\{b_n\}$. Does $\sum b_n$ converge? YES \rightarrow Is $0 \leq a_n \leq b_n$? YES $\rightarrow \sum a_n$ Converges
NO \rightarrow Is $0 \leq b_n \leq a_n$? YES $\rightarrow \sum a_n$ Diverges

LIMIT COMPARISON TEST

Pick $\{b_n\}$. Does $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c > 0$ c finite & $a_n, b_n > 0$? YES \rightarrow Does $\sum_{n=1}^{\infty} b_n$ converge? YES $\rightarrow \sum a_n$ Converges
NO $\rightarrow \sum a_n$ Diverges

INTEGRAL TEST

Does $a_n = f(n), f(x)$ is continuous, positive & decreasing on $[a, \infty)$? YES \rightarrow Does $\int_a^{\infty} f(x) dx$ converge? YES $\rightarrow \sum_{n=a}^{\infty} a_n$ Converges
NO $\rightarrow \sum a_n$ Diverges

RATIO TEST

Is $\lim_{n \rightarrow \infty} |a_{n+1}/a_n| \neq 1$? YES \rightarrow Is $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$? YES $\rightarrow \sum a_n$ Abs. Conv.
NO $\rightarrow \sum a_n$ Diverges

ROOT TEST

Is $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} \neq 1$? YES \rightarrow Is $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} < 1$? YES $\rightarrow \sum a_n$ Abs. Conv.
NO $\rightarrow \sum a_n$ Diverges

$$1. \sum_{n=1}^{\infty} \frac{n^2 - 1}{n^2 + n}$$

$$2. \sum_{n=1}^{\infty} \frac{n - 1}{n^2 + n}$$

$$3. \sum_{n=1}^{\infty} \frac{1}{n^2 + n}$$

$$4. \sum_{n=1}^{\infty} (-1)^{n-1} \frac{n - 1}{n^2 + n}$$

$$5. \sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$$

$$6. \sum_{n=1}^{\infty} \left(\frac{3n}{1 + 8n} \right)^n$$

$$7. \sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln(n)}}$$

$$8. \sum_{k=1}^{\infty} \frac{2^k k!}{(k + 2)!}$$

$$9. \sum_{k=1}^{\infty} k^2 e^{-k}$$

$$10. \sum_{n=1}^{\infty} n^2 e^{-n^3}$$

$$11. \sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n \ln(n)}$$

$$12. \sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 25}$$

$$13. \sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$$

$$14. \sum_{n=1}^{\infty} \sin(n)$$

$$15. \sum_{n=0}^{\infty} \frac{n!}{2 \cdot 5 \cdot 8 \cdots (3n + 2)}$$

$$16. \sum_{n=1}^{\infty} \frac{n^2 + 1}{n^3 + 1}$$

$$17. \sum_{n=1}^{\infty} (-1)^n 2^{1/n}$$

$$18. \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n} - 1}$$

$$19. \sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{\sqrt{n}}$$

$$20. \sum_{k=1}^{\infty} \frac{k + 5}{5^k}$$

$$21. \sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n}$$

$$22. \sum_{n=1}^{\infty} \frac{\sqrt{n^2 - 1}}{n^3 + 2n^2 + 5}$$

$$23. \sum_{n=1}^{\infty} \tan(1/n)$$

$$24. \sum_{n=1}^{\infty} \frac{\cos(n/2)}{n^2 + 4n}$$

$$25. \sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$$

$$26. \sum_{n=1}^{\infty} \frac{n^2 + 1}{5^n}$$

$$27. \sum_{k=1}^{\infty} \frac{k \ln(k)}{(k + 1)^3}$$

$$28. \sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$$

$$29. \sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{n\sqrt{n}}$$

$$30. \sum_{j=1}^{\infty} (-1)^j \frac{\sqrt{j}}{j + 5}$$

$$31. \sum_{k=1}^{\infty} \frac{5^k}{3^k + 4^k}$$

$$32. \sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2n}}$$

$$33. \sum_{n=1}^{\infty} \frac{\sin(1/n)}{\sqrt{n}}$$

$$34. \sum_{n=1}^{\infty} \frac{1}{n + n \cos^2(n)}$$

$$35. \sum_{n=1}^{\infty} \left(\frac{n}{n + 1} \right)^{n^2}$$

$$36. \sum_{n=2}^{\infty} \frac{1}{(\ln(n))^{\ln(n)}}$$

$$37. \sum_{n=1}^{\infty} (\sqrt[n]{2} - 1)^n$$

$$38. \sum_{n=1}^{\infty} (\sqrt[n]{2} - 1)$$