MATH 412 HW 10

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1. Let G be a group. The *center* of G, denoted $Z(G)$, is defined by
$$Z(G) = \{z \in G : zx = xz \text{ for all } x \in G\}$$
In words, the center is the set of elements of G that commute with every element of G. Prove that $Z(G)$ is a normal subgroup of G. (If you got this right on the quiz, you can skip this problem.)

Solution:

2. Let G be an abelian group and let T be the set of elements of finite order. Show that T is a subgroup of G.

Solution:

3. Let G be a group and let a and $b \in G$. Show that if $ab \in Z(G)$ then $ba \in Z(G)$.

Solution:

4. Let U_n be the units of $\mathbb{Z}_n$. Show that U_5 and U_{10} are both isomorphic to a cyclic group of order 4.

Solution:

5. Let H be a subgroup of a group G and let $a \in G$. Show that aHa^{-1} is also a subgroup of G and that H and aHa^{-1} are isomorphic.

Solution:

6. Find the order of σ^{100}, where σ is the permutation
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 7 & 8 & 9 & 4 & 5 & 2 & 1 & 6 \end{pmatrix}.$$
Hint: Write σ as a product of disjoint cycles.

Solution: